FÖRSLAG PÅ LÖSNING TILL TENTAMEN I MATEMATIK BASKURS SF1612, MATEMATIK I FÖR CL SF1623.

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1. $3\sqrt{x+13} = x+9 \qquad x+13 \geq 0 \Leftrightarrow x \geq -13.$

$9(x+13) = (x+9)^2$

$9x + 117 = x^2 + 18x + 81$

$x^2 + 9x - 36 = 0$

$x = -\frac{9}{2} \pm \sqrt{\frac{81}{4} + \frac{144}{4}} = -\frac{9}{2} \pm \sqrt{\frac{225}{4}}$

$x = -\frac{9}{2} \pm \frac{15}{2}, \quad x_1 = \frac{6}{2} = 3 \quad x_2 = \frac{-24}{2} = -12$

KOLL $x_1 = 3 \quad VL = 3\sqrt{3+13} = 3\sqrt{16} = 3 \cdot 4 = 12$, $HL = 3+9 = 12$ > SAT

$x_2 = -12 \quad VL = 3\sqrt{-12+13} = 3\sqrt{1} = 3$, $HL = -12+9 = -3$ > SAT. EJ

SVAR: $x = 3$

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2. $$\left(x^4 + \frac{a}{x^2}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (x^4)^k \cdot \left(\frac{a}{x^2}\right)^{12-k}$$

$$(x^4)^k \left(\frac{a}{x^2}\right)^{12-k} = x^{4k} \cdot \frac{a^{12-k}}{(x^2)^{12-k}} = x^{4k} \cdot a^{12-k} \cdot x^{2k-24} =$$

$$= x^{6k-24} \cdot a^{12-k}$$

KONSTANTA TERMEN HITTAS DÅ $6k - 24 = 0 \Leftrightarrow k = 4$

DESS KOEFF: $\binom{12}{4} a^{12-4} = \frac{12!}{8!4!} a^8 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} a^8 =$

$= 11 \cdot 5 \cdot 9 \, a^8 = 495 a^8$ SVAR: $495 a^8$

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3. $\sin 2x = 2\cos 2x \cos x$

$2 \sin x \cos x = 2 \cos 2x \cos x$

$2 \sin x \cos x - 2\cos 2x \cos x = 0$

$2 \cos x (\sin x - \cos 2x) = 0$

1) $2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2} + n \cdot \pi$

FORTS.

3. FORTS.

a)

$$\sin x - \cos 2x = 0$$

$$\sin x - (1 - 2\sin^2 x) = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

SÄTT $\sin x = t$

$$2t^2 + t - 1 = 0$$

$$t^2 + \frac{t}{2} - \frac{1}{2} = 0 \quad \nearrow$$

$$t = -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}} = -\frac{1}{4} \pm \sqrt{\frac{9}{16}}$$

$$t = -\frac{1}{4} \pm \frac{3}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + n \cdot 2\pi$$

$$x = \pi - \frac{\pi}{6} + n \cdot 2\pi = \frac{5\pi}{6} + n \cdot 2\pi$$

$$\sin x = -1$$

$$x = -\frac{\pi}{2} + n \cdot 2\pi$$

SVAR: $x = \frac{\pi}{2} + n \cdot \pi$, $x = \frac{\pi}{6} + n\,2\pi$, $x = \frac{5\pi}{6} + n \cdot 2\pi$

4.

$$f(x) = x - \frac{1}{x}, \quad x > 0$$

$$g(x) = \frac{x + \sqrt{x^2+4}}{2}$$

a)

$$f(g(x)) = \frac{x + \sqrt{x^2+4}}{2} - \frac{1}{\frac{x + \sqrt{x^2+4}}{2}} =$$

$$= \frac{x + \sqrt{x^2+4}}{2} - \frac{2}{x + \sqrt{x^2+4}} = \frac{(x + \sqrt{x^2+4})^2 - 2^2}{2(x + \sqrt{x^2+4})} =$$

$$= \frac{x^2 + 2x\sqrt{x^2+4} + x^2 + 4 - 4}{2(x + \sqrt{x^2+4})} = \frac{2(x^2 + x\sqrt{x^2+4})}{2(x + \sqrt{x^2+4})} =$$

$$= \frac{x\,(x + \sqrt{x^2+4})}{x + \sqrt{x^2+4}} = x$$

b)

$$g(f(x)) = \frac{x - \frac{1}{x} + \sqrt{(x - \frac{1}{x})^2 + 4}}{2} = \frac{x - \frac{1}{x} + \sqrt{x^2 - 2 + \frac{1}{x^2} + 4}}{2}$$

$$= \frac{x - \frac{1}{x} + \sqrt{x^2 + 2 + \frac{1}{x^2}}}{2} = \frac{x - \frac{1}{x} + \sqrt{(x + \frac{1}{x})^2}}{2} = \frac{x - \frac{1}{x} + |x + \frac{1}{x}|}{2}$$

$$= \frac{x - \frac{1}{x} + x + \frac{1}{x}}{2} = \frac{2x}{2} = x$$

FORTS.

4c) DE ÄR VARANDRAS INVERSER. DVS $f^{-1}(x) = g(x)$ OCH $g^{-1}(x) = f(x)$.

5.a) $p(x) = x^3 + kx + 6$ $\quad$ $p(x)$ DELBART MED $x-2$

$\Leftrightarrow \quad p(2) = 0 \qquad p(2) = 8 + 2k + 6 = 14 + 2k$

$p(2) = 0 \quad \Rightarrow \quad 14 + 2k = 0 \quad \Leftrightarrow \quad k = -7$

$p(x) = x^3 - 7x + 6$ $\qquad$ <u>SVAR</u>! $k = -7$

b) $x^3 - 7x + 6 = q(x)(x-2) \qquad q(x) = \dfrac{x^3 - 7x + 6}{x-2} = x^2 + 2x - 3$

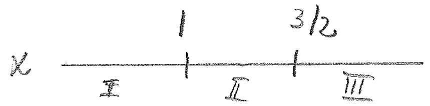
$$
\begin{array}{l|l}
\multicolumn{1}{l}{x^2 + 2x - 3} & \\
\hline
x^3 \qquad\quad - 7x + 6 & x - 2 \\
-x^3 \overset{+}{(-)} 2x^2 & \\
\cline{1-1}
\quad 2x^2 - 7x & \\
\quad -2x^2 \overset{+}{(-)} 4x & \\
\cline{1-1}
\qquad -3x + 6 & \\
\qquad -3x + 6 & \\
\cline{1-1}
\qquad\qquad 0 &
\end{array}
$$

$x^3 - 7x + 6 = (x^2 + 2x - 3)(x-2) = (x+3)(x-1)(x-2)$

<u>SVAR</u>! $x^3 - 7x + 6 = (x-1)(x-2)(x+3)$

6. $x + |2x - 3| \geq |x - 1| + 2$

$|2x - 3| = \begin{cases} 2x - 3 & x \geq \frac{3}{2} \\ -(2x-3) & x < \frac{3}{2} \end{cases} \qquad |x - 1| = \begin{cases} x - 1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$



TRE OLIKA FALL!

I $\quad x < 1$: $\quad x - (2x - 3) \geq -(x-1) + 2$

$x - 2x + 3 \geq -x + 1 + 2$

$-x + 3 \geq -x + 3$ $\quad$ ALLTID SANT! $\quad$; $\underline{\underline{x < 1}}$

II $\quad 1 \leq x < \frac{3}{2}$: $\quad x - (2x-3) \geq x - 1 + 2$

$x - 2x + 3 \geq x + 1$

$-x + 3 \geq x + 1$

$2 \geq 2x \Leftrightarrow x \leq 1$ $\quad$; $\quad x = 1$

III $\quad x \geq \frac{3}{2}$ $\quad x + 2x - 3 \geq x - 1 + 2$

$3x - 3 \geq x + 1$

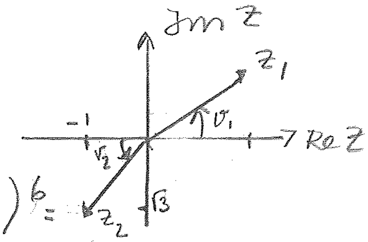
$2x \geq 4$, $\quad x \geq 2$

<u>SVAR</u>!

$x \leq 1$ ELLER $x \geq 2$

7.

$$\frac{(\sqrt{6}+i\sqrt{2})^6}{(-1-i\sqrt{3})^{15}} = \frac{z_1}{z_2}$$



$\tan v_1 = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}}$

$v_1 = \frac{\pi}{6} + n\cdot\pi$

$|z_1| = \sqrt{6+2} = \sqrt{8}$

$$z_1 = (\sqrt{6}+i\sqrt{2})^6 = \left(\sqrt{8}\, e^{i\cdot\frac{\pi}{6}}\right)^6 =$$

$$= (2\sqrt{2})^6 \cdot e^{i\cdot\pi} = 2^6\cdot 2^3 \cdot e^{\pi i} = 2^9\cdot(-1)$$

$\tan v_2 = \frac{\sqrt{3}}{1}$ $\quad v_2 = \frac{\pi}{3} + n\cdot\pi$

$$z_2 = \left(2\, e^{i\cdot\frac{4\pi}{3}}\right)^{15} = 2^{15}\cdot e^{20\pi i} = 2^{15}\cdot 1$$

$|z_2| = \sqrt{1+3} = 2$

$\arg z_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\frac{z_1}{z_2} = \frac{-2^9}{2^{15}} = -\frac{1}{2^6}$$

SVAR: $-\frac{1}{2^6}$

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8. a)

$$A(x) = 8\cdot 14 - \frac{8\cdot x}{2} - \frac{(14-x)\cdot x}{2} - \frac{(8-x)\cdot 14}{2} = 112 - 4x - 7x + \frac{x^2}{2} -$$

$$- 56 + 7x = \frac{x^2}{2} - 4x + 56$$

b) DEFINITIONSMÄNGD: $0 \le x \le 8$

c)

$$\frac{x^2}{2} - 4x + 56 = \frac{1}{2}(x^2 - 8x + 112) = \frac{1}{2}\left[(x-4)^2 + 112 - 16\right] =$$

$$= \frac{1}{2}\left((x-4)^2 + 96\right) = \frac{(x-4)^2}{2} + 48$$

MINSTA VÄRDE: 48
STÖRSTA VÄRDE: 56

VÄRDEMÄNGD: $48 \le A(x) \le 56$

SVAR: a) $A(x) = \frac{x^2}{2} - 4x + 56$ b) D_A: $0 \le x \le 8$ c) V_A: $48 \le A(x) \le 56$
MINSTA V: 48
STÖRSTA V: 56

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9) $P(n)$: $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1}\cdot n^2 = (-1)^{n+1}\cdot\frac{n(n+1)}{2}$ $\quad n \geq 1$

1) $n=1$ $\quad VL = 1^2 = 1$ $\quad HL = (-1)^2\cdot\frac{1\cdot 2}{2} = 1$

$VL = HL$. ANTAG $P(n)$ SANN.

VISA $P(n+1)$:

$$1^2 - 2^2 + 3^2 + \dots + (-1)^{n-1}\cdot n^2 + (-1)^{n+1-1}\cdot(n+1)^2 = |\text{ENL } P(n)| =$$

$$= (-1)^{n+1}\cdot\frac{n(n+1)}{2} + (-1)^n(n+1)^2 = -(-1)^n\frac{n}{2}(n+1) + (-1)^n(n+1)^2 =$$

$$= (-1)^n(n+1)\left(-\frac{n}{2} + n + 1\right) = (-1)^n(n+1)\left(\frac{n}{2}+1\right) = (-1)^n(n+1)\left(\frac{n+2}{2}\right) =$$

$$= 1\cdot(-1)^n\frac{(n+1)(n+1+1)}{2} = (-1)(-1)(-1)^n\frac{(n+1)(n+1+1)}{2} = (-1)^{n+2}\frac{(n+1)(n+1+1)}{2}$$

$P(n) \Rightarrow P(n+1)$ SANN.

$\Rightarrow P(n)$ SANN FÖR ALLA $n \geq 1$ ENLIGT INDUKTIONSPRINCIPEN.