FÖRSLAG TILL LÖSNING 5B1123 (HELA KURSEN)
MATEMATIK II FÖR LÄRARE, 2004-08-17

1. $\bar{v}_1 = (2,-3,1)$ $\bar{v}_2 = (4,1,1)$ $\bar{v}_3 = (0,-7,1)$

UNDERSÖK OM $\bar{v}_1$, $\bar{v}_2$ OCH $\bar{v}_3$ ÄR LINJÄRT OBEROENDE

$$\begin{vmatrix} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 28 + 14 + 12 = 28 - 28 = 0$$

$\therefore$ VEKTORERNA ÄR LINJÄRT BEROENDE OCH KAN EJ UTGÖRA EN BAS. SVAR: NEJ
(t.ex $\bar{v}_3 = 2\bar{v}_1 - \bar{v}_2$)

2. Π_1 : $3x - 4y + z = 1$ $\bar{n}_{\Pi_1} = (3,-4,1)$

Π_2 : $6x - 8y + 2z = 3$ $\bar{n}_{\Pi_2} = (6,-8,2) = 2(3,-4,1)$

$2\bar{n}_{\Pi_1} = \bar{n}_{\Pi_2}$ $\therefore$ NORMALERNA ÄR PARALLELLA

$\Rightarrow$ PLANEN ÄR PARALLELLA. SÖK EN PUNKT PÅ Π_1 t.ex $(-1,-1,0) = P_0$

AVSTÅNDET P_0 TILL Π_2 ÄR

$$d = \frac{|6\cdot(-1) - 8(-1) + 2\cdot 0 - 3|}{\sqrt{6^2 + (-8)^2 + 2^2}} = \frac{|-6 + 8 - 3|}{\sqrt{104}} = \frac{1}{2\sqrt{26}}$$

SVAR: $\frac{1}{2\sqrt{26}}$ l.e

3. $f(x,y,z) = xy - z$ $P_0 = (-2,-3,6)$

$$\bar{n}_{\Pi} = \text{grad } F = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (y, x, -1)$$

$$\bar{n}_{\Pi}(-2,-3,6) = (-3,-2,-1) = -(3,2,1)$$

Π : $\bar{n}_{\Pi} \cdot (x+2, y+3, z-6) = 0$

$(3,2,1)(x+2, y+3, z-6) = 0$

$3(x+2) + 2(y+3) + z - 6 = 0$

$3x + 2y + z + 6 = 0$ SVAR: $3x + 2y + z + 6 = 0$

4. $\sum_{n=1}^{\infty} \frac{3\sqrt{n}+2}{n^2-4n}$ $\qquad a_n = \frac{3\sqrt{n}+2}{n^2-4n}$

LÅT $b_n = \frac{1}{n\sqrt{n}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{3\sqrt{n}+2}{n^2-4n}}{\frac{1}{n\sqrt{n}}} = \lim_{n\to\infty} \frac{3\sqrt{n}+2}{n^2-4n} \cdot \frac{n\sqrt{n}}{1} =$$

$$= \lim_{n\to\infty} \frac{3n^2+2n\sqrt{n}}{n^2-4n} = \lim_{n\to\infty} \frac{n^2\left(3+\frac{2}{\sqrt{n}}\right)}{n^2\left(1-\frac{4}{n}\right)} =$$

$$= \lim_{n\to\infty} \frac{3+\frac{2}{\sqrt{n}}}{1-\frac{4}{n}} = 3.$$ $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ ÄR KONV TY

$\sum \frac{1}{n^a}$ KONV. OM $a > 1$. $\Rightarrow \sum_{n=1}^{\infty} \frac{3\sqrt{n}+2}{n^2-4n}$

ÄR KONV. ENL JÄMFÖRELSE PRINCIPEN.

<u>SVAR</u>: KONVERGENT

5.

$$z^2 - 2z + iz + 3 - i = 0$$

$$z^2 + (i-2)z + 3 - i = 0$$

$$z = \frac{2-i}{2} \pm \sqrt{\left(\frac{2-i}{2}\right)^2 - 3 + i}$$

$$\left(\frac{2-i}{2}\right)^2 - 3 + i = (a+ib)^2 = a^2 - b^2 + i \cdot 2ab$$

$$\frac{4-4i+i^2-12+4i}{4} = a^2 - b^2 + i \cdot 2ab$$

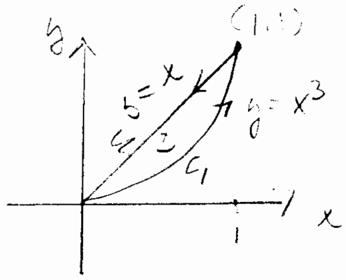
$$\begin{cases} -\frac{9}{4} = a^2 - b^2 \\ 2ab = 0 \end{cases} \Rightarrow a = 0 \text{ eller } b = 0$$

$a = 0 \Rightarrow b = \pm\frac{3}{2}$, $b = 0 \Rightarrow a = \pm\frac{3}{2}i$ (ej def. ty a reell)

$z = \frac{2-i}{2} \pm \frac{3}{2}i = 1 - \frac{i}{2} \pm \frac{3}{2}i$ $\quad \begin{cases} z_1 = 1+i \\ z_2 = 1-2i \end{cases}$

<u>SVAR:</u> $z_1 = 1+i$, $z_2 = 1-2i$

6. $\int_C y^3\,dx + (x^3 + 3xy^2)\,dy$



SLUTET OMRÅDE, INGA SINGULARITETER. GREENS FORMEL KAN ANVÄNDAS

$$\frac{\partial P}{\partial y} = 3y^2 \qquad \frac{\partial Q}{\partial x} = 3x^2 + 3y^2$$

$$\int_S = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy = \iint_R 3x^2\,dx\,dy =$$

$$= \int_0^1 \int_{x^3}^{x} 3x^2\,dy\,dx = \int_0^1 \Big[3x^2 y\Big]_{x^3}^{x} dx = \int_0^1 (3x^3 - 3x^5)\,dx =$$

$$= \left[\frac{3x^4}{4} - \frac{3x^6}{6}\right]_0^1 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}.$$ <u>SVAR</u>: $\frac{1}{4}$

(ALT. INTEGRERA LÄNGS C_1 OCH C_2)

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7. $z = 4 - x^2 - y^2 \qquad P_0 = (1,1,2)$

BESTÄM $\frac{df}{d\bar{e}}$ i P_0. NORDOST $= \bar{e} = (1,1)$ $\quad |\bar{e}| = \sqrt{2}$

$$\frac{df}{d\bar{e}} = \text{grad}\, z \cdot \frac{\bar{e}}{|\bar{e}|} \quad \text{i } P_0$$

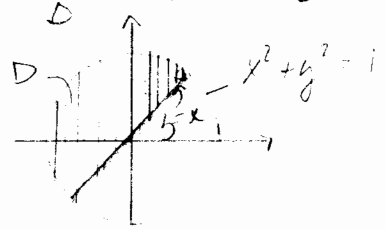
$$\text{grad}\, z = (-2x, -2y) \qquad \text{grad}\, z\,(1,1) = (-2,-2)$$

$$\frac{df}{d\bar{e}} = (-2,-2)\cdot\frac{(1,1)}{\sqrt{2}} = \frac{1}{\sqrt{2}}(-2-2) = \frac{-4}{\sqrt{2}} = -2\sqrt{2} < 0$$

!! NEDFÖR BERGET.

<u>SVAR</u>: NEDFÖR BERGET

8. $I = \iint_D x^2 y \, dx dy \qquad D = \{(x,y) : x^2+y^2 \le 1,\ y \ge x\}$



$\iint x^2 y \, dx dy = \left| \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ dx dy = r\,dr\,d\theta \\ 0 \le r \le 1 \\ \frac{\pi}{4} \le \theta \le \frac{5\pi}{4} \end{array} \right| =$

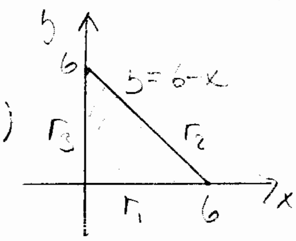
$= \int_{\pi/4}^{5\pi/4} \int_0^1 r^2 \cos^2\theta \cdot r\sin\theta \cdot r \, dr \, d\theta = \int_{\pi/4}^{5\pi/4} \cos^2\theta \sin\theta \, d\theta \cdot \int_0^1 r^4 \, dr$

$= \left[-\frac{\cos^3\theta}{3} \right]_{\pi/4}^{5\pi/4} \cdot \left[\frac{r^5}{5} \right]_0^1 = \frac{1}{3}\left(-\left(-\frac{1}{\sqrt{2}}\right)^3 - \left(-\frac{1}{\sqrt{2}}\right)^3 \right) \cdot \frac{1}{5} =$

$= \frac{1}{3}\left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) \cdot \frac{1}{5} = \frac{1}{15\sqrt{2}}$ Svar: $\frac{1}{15\sqrt{2}} = \frac{\sqrt{2}}{30}$

9. $f(x,y) = x^2y^2 + x^3y - 4x^2y \qquad x \ge 0,\ y \ge 0 \quad x+y \le 6$

INRE STAT PUNKTER!



I) $f'_x = 2xy^2 + 3x^2y - 8xy = xy(2y+3x-8)$

II) $f'_y = 2x^2y + x^3 - 4x^2 = x^2(2y+x-4)$

$f'_x = 0 \Rightarrow xy(2y+3x-8) = 0 \qquad x=0,\ y=0$ (randpkt)

$f'_y = 0 \Rightarrow x^2(2y+x-4) = 0 \qquad x = 0 \Rightarrow y = 8$ [illegible]

$f(0,y) = \underline{0} \qquad y = 0 \Leftrightarrow x = 0$ eller $x = 4$

II) $2y+x-4 = 0 \Leftrightarrow x = 4-2y$ INSATT I I $2y + 3(4-2y) - 8 = 0$

$2y + 12 - 6y - 8 = 0 \Leftrightarrow -4y = -4,\ y = 1 \Rightarrow x = 2$

$f(2,1) = 4 + 8 - 4 \cdot 4 = 12 - 16 = \underline{\underline{-4}}$

RAND: $\Gamma_1: y = 0 \Rightarrow f(x,0) = 0$

$\Gamma_2: y = 6-x \Rightarrow f(x,6-x) = x^2(6-x)^2 + x^3(6-x) - 4x^2(6-x)$

$= g(x)$. $g'(x) = 2x(6-x)^2 - x^2 \cdot 2(6-x)(-1) + 3x^2(6-x) + x^3(-1) - 8x(6-x) + 4x^2$

$36-12x+x^2$

$= $ [illegible] $- 12x^2 + $ [illegible] $+ 18x^2$ [illegible] $x^3 - 48x + 8x^2 + 4x^2 =$

$= -6x^2 + 24x = 6x(-x+4)$, $g'(x) = 0 \Leftrightarrow x = 0,\ x = 4$

$g(0) = 0 \quad g(4) = 16 \cdot 4 + 64 \cdot 2 - 64 \cdot 2 = 16 \cdot 4 = \underline{\underline{64}}$

FORTS.

9) FORTS.

r_3 $x = 0$ $f(0, y) = 0$

HÖRN $f(0,0) = 0$, $f(6,0) = 0$ $f(0,6) = 0$

<u>SVAR</u>: STÖRSTA VÄRDE: $g(4) = f(4,2) = 64$
MINSTA VÄRDE: $f(2,1) = -4$

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10. $A = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$ KAR. EKV $\begin{vmatrix} 5-\lambda & -6 \\ 3 & -4-\lambda \end{vmatrix} = 0$

$(5-\lambda)(-4-\lambda) + 18 = 0$

$-20 - 5\lambda + 4\lambda + \lambda^2 + 18 = 0$

$\lambda^2 - \lambda - 2 = 0$

$\lambda = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}$

$\lambda = \frac{1}{2} \pm \frac{3}{2}$

$\lambda_{a1} = -1$
$\lambda_{a2} = 2$ } EGENVÄRDEN

EGENVEKTORER:

$\lambda_1 = -1$ $\left(\begin{array}{cc|c} 6 & -6 & 0 \\ 3 & -3 & 0 \end{array}\right)$ $6x - 6y = 0$, $x = y$ $\bar{v}_1 = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$ $\left(\begin{array}{cc|c} 3 & -6 & 0 \\ 3 & -6 & 0 \end{array}\right)$ $3x - 6y = 0$, $x = 2y$ $\bar{v}_2 = t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

<u>SVAR</u> a) $\lambda_{a1} = -1$ $\bar{v}_1 = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\lambda_{a2} = 2$ $\bar{v}_2 = t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

b) $B\bar{v} = \lambda\bar{v} \Rightarrow B^{-1} \cdot B\bar{v} = B^{-1}\lambda\bar{v}$

$\bar{v} = \lambda B^{-1}\bar{v}$

$B^{-1}\bar{v} = \frac{1}{\lambda}\bar{v}$

<u>SVAR</u>: B^{-1} HAR EGENVÄRDEN $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$

c) $C\bar{v} = \lambda\bar{v} \Rightarrow C^2\bar{v} = C \cdot C\bar{v} = C \cdot \lambda\bar{v} = \lambda C\bar{v} = \lambda \cdot \lambda\bar{v}$

$= \lambda^2\bar{v}$!! $C^2\bar{v} = \lambda^2\bar{v}$

<u>SVAR</u>: C^2 HAR EGENVÄRDEN $\lambda_1^2, \lambda_2^2 \ldots \lambda_n^2$.

FORTS.

10 FORTS.

10 d) $A = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$

$$A^{-1} = \frac{1}{-20+18}\begin{pmatrix} -4 & 6 \\ -3 & 5 \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -4 & 6 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ \frac{3}{2} & -\frac{5}{2} \end{pmatrix}$$

EGENVÄRDEN TILL A^{-1}!

$$\begin{vmatrix} 2-\lambda & -3 \\ \frac{3}{2} & -\frac{5}{2}-\lambda \end{vmatrix} = 0 \qquad (2-\lambda)\left(-\frac{5}{2}-\lambda\right) + \frac{9}{2} = 0$$

$$-5 - 2\lambda + \frac{5\lambda}{2} + \lambda^2 + \frac{9}{2} = 0$$

$$\lambda^2 + \frac{\lambda}{2} - \frac{1}{2} = 0 \qquad \lambda = -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}} = -\frac{1}{4} \pm \frac{3}{4}$$

$$\lambda_{A^{-1}_1} = -1 = \frac{1}{\lambda_{a1}} \qquad \lambda_{A^{-1}_2} = \frac{1}{2} = \frac{1}{\lambda_{a2}}$$

$$A^2 = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ 3 & -2 \end{pmatrix}$$

EGENVÄRDEN TILL A^2!

$$\begin{vmatrix} 7-\lambda & -6 \\ 3 & -2-\lambda \end{vmatrix} = 0 \qquad (7-\lambda)(-2-\lambda) + 18 = 0$$

$$-14 - 7\lambda + 2\lambda - \lambda^2 + 18 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}} = \frac{5}{2} \pm \frac{3}{2}$$

$$\lambda_{A^2_1} = 1 = (\lambda_{a1})^2 \qquad \lambda_{A^2_2} = 4 = (\lambda_{a2})^2$$