FÖRSLAG TILL LÖSNING MATEMATIK II FÖR CL

5B1123. 2005-08-27

1a)

$$\begin{cases} x-4y+z=2 \\ 2x-5y+z=4 \\ -3x+7y-z=1 \end{cases} \Leftrightarrow \begin{pmatrix} 1 & -4 & 1 \\ 2 & -5 & 1 \\ -3 & 7 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \Leftrightarrow A\bar{x}=\bar{b}$$

i)

$$\left(\begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 2 & -5 & 1 & 4 \\ -3 & 7 & -1 & 1 \end{array}\right) (-2)\ (3) \sim \left(\begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 0 & 3 & -1 & 0 \\ 0 & -5 & 2 & 7 \end{array}\right) \begin{matrix} \cdot 5 \\ \cdot 3 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 0 & 15 & -5 & 0 \\ 0 & -15 & 6 & 21 \end{array}\right) (1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 0 & 15 & -5 & 0 \\ 0 & 0 & 1 & 21 \end{array}\right) (5)\ (-1) \sim \left(\begin{array}{ccc|c} 1 & -4 & 0 & -19 \\ 0 & 15 & 0 & 105 \\ 0 & 0 & 1 & 21 \end{array}\right) \cdot \tfrac{1}{15} \sim \left(\begin{array}{ccc|c} 1 & -4 & 0 & -19 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 21 \end{array}\right) (4)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 21 \end{array}\right) \Leftrightarrow \begin{matrix} x=9 \\ y=7 \\ z=21 \end{matrix} \qquad \text{SVAR: } \begin{cases} x=9 \\ y=7 \\ z=21 \end{cases}$$

ii)

$$B=\begin{pmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 5 & 3 \end{pmatrix} \qquad A=\begin{pmatrix} 1 & -4 & 1 \\ 2 & -5 & 1 \\ -3 & 7 & -1 \end{pmatrix}$$

B ÄR INVERS TILL A OM $AB=BA=E$

$$BA=\begin{pmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 5 & 3 \end{pmatrix}\begin{pmatrix} 1 & -4 & 1 \\ 2 & -5 & 1 \\ -3 & 7 & -1 \end{pmatrix} = \begin{pmatrix} -2+6-3 & 8-15+7 & -2+3-1 \\ -1+4-3 & +4-10+7 & -1+2-1 \\ -1+10-9 & 4-25+21 & -1+5-3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -4 & 1 \\ 2 & -5 & 1 \\ -3 & 7 & -1 \end{pmatrix}\begin{pmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 5 & 3 \end{pmatrix} = AB$$

$$A\bar{x}=\bar{b} \Leftrightarrow A^{-1}\cdot A\bar{x} = A^{-1}\cdot\bar{b} \Leftrightarrow \bar{x}=A^{-1}\cdot\bar{b} \Leftrightarrow \bar{x}=B\cdot\bar{b}$$

$$\bar{x}=\begin{pmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 5 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -4+12+1 \\ -2+8+1 \\ -2+20+3 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \\ 21 \end{pmatrix} \quad \therefore \quad \begin{matrix} x=9 \\ y=7 \\ z=21 \end{matrix}$$

$$\text{SVAR: } \begin{cases} x=9 \\ y=7 \\ z=21 \end{cases}$$

1b) $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 0 & -2 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ 2 & 0 & -2 \end{pmatrix}$

$\det AB = \det A \cdot \det B$

$\det A = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 0 & -2 & 5 \end{vmatrix} = 5 + 0 - 12 - 0 + 8 + 15 = 28 - 12 = 16$

$\det B = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ 2 & 0 & -2 \end{vmatrix} = 2 - 16 + 0 + 6 + 0 + 0 = -8$

$\det AB = 16 \cdot (-8) = -128$ SVAR: -128

2a) $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$

EGENVÄRDEN: $\begin{vmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{vmatrix} = 0 \iff$

$(1-\lambda)(2-\lambda)(-1-\lambda) + 2 + 12 - 8(2-\lambda) + (1-\lambda) + 3(-1-\lambda) = 0$

$(1-\lambda)(2-\lambda)(-1-\lambda) + 14 - 16 + 8\lambda + 1 - \lambda - 3 - 3\lambda = 0$

$(1-\lambda)(2-\lambda)(-1-\lambda) - 4 + 4\lambda = 0 \iff$

$-(1-\lambda)(2-\lambda)(1+\lambda) - 4(1-\lambda) = 0 \iff (1-\lambda)(-4 - (1+\lambda)(2-\lambda)) = 0$

i) $1 - \lambda = 0$, $\lambda = 1$

ii) $-4 - (1+\lambda)(2-\lambda) = 0 \iff$
$-4 - (2 + \lambda - \lambda^2) = 0 \iff$
$\lambda^2 - \lambda - 6 = 0 \iff (\lambda+2)(\lambda-3) = 0$
$\lambda = 3$, $\lambda = -2$

EGENVÄRDEN: $\lambda_1 = -2$, $\lambda_2 = 1$, $\lambda_3 = 3$

EGENVEKTORER: $\lambda_1 = -2$ GER $\left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 0 & 5 & -5 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \tfrac{1}{5}$

$\sim \left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 2 & 0 & 0 \end{array}\right)$ $x = -z$, $y = z$ $\bar{v}_1 = t\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 1$ GER $\left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 1 & -1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 \end{array}\right) \begin{matrix} \\ \tfrac{1}{3} \\ \tfrac{1}{2} \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right)$

$\begin{cases} x = -z \\ y = 4z \end{cases}$ $\bar{v}_2 = t\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$

FORTS

FORTS 2a)

$\lambda_3 = 3$ GER $\left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & 1 & -4 & 0 \end{array}\right) \begin{matrix} (-1) & (1) \end{matrix} \sim \left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ +5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) (-\tfrac{1}{5}) \sim \left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right)$

$x = z \Rightarrow y = 4z - 2x = 4z - 2z = 2z \qquad \bar{v}_3 = t\begin{pmatrix}1\\2\\1\end{pmatrix}$

SVAR! $\lambda_1 = -2$, $\bar{v}_1 = t\begin{pmatrix}-1\\1\\1\end{pmatrix}$; $\lambda_2 = 1$, $\bar{v}_2 = t\begin{pmatrix}-1\\4\\1\end{pmatrix}$;
$\lambda_3 = 3$, $\bar{v}_3 = t\begin{pmatrix}1\\2\\1\end{pmatrix}$

2b) $\bar{v}_1 = (0,0,1)$, $\bar{v}_2 = (7,4,11)$, $\bar{v}_3 = (5,3,13)$
OM VEKTORERNA ÄR LINJÄRT OBEROENDE UTGÖR DE EN BAS I R^3. UNDERSÖK DETERMINANTEN

$$\begin{vmatrix} 0 & 7 & 5 \\ 0 & 4 & 3 \\ 1 & 11 & 13 \end{vmatrix} = 21 - 20 = 1 \neq 0 \quad \therefore \text{ LINJ. OBEROENDE}$$

SVAR! DE TRE VEKTORERNA BILDAR EN BAS I R^3.

3a) $\sum_{n=0}^{\infty} \frac{n^2 \cdot 2^{n+1}}{3^n} \qquad a_n = \frac{n^2 \cdot 2^{n+1}}{3^n}$

KVOTKRIT. GER $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^2 \cdot 2^{n+2}}{3^{n+1}}}{\frac{n^2 \cdot 2^{n+1}}{3^n}}$

$$= \lim_{n\to\infty} \frac{(n+1)^2 \cdot 2^n \cdot 2^2}{3^n \cdot 3} \cdot \frac{3^n}{n^2 \cdot 2^n \cdot 2} =$$

$$= \lim_{n\to\infty} \frac{(n+1)^2 \cdot 2}{n^2 \cdot 3} = \lim_{n\to\infty} \frac{n^2\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot 2}{n^2 \cdot 3} =$$

$$= \lim_{n\to\infty} \frac{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)}{3} \cdot 2 = \frac{2}{3} < 1 \quad \therefore \text{ KONVERGENT}$$

SVAR! KONVERGENT

3b)

$$v(x,y,z) = \ln r \qquad r = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{\frac{1}{2}}$$

$$v = \ln (x^2+y^2+z^2)^{\frac{1}{2}} = \frac{1}{2} \ln (x^2+y^2+z^2)$$

$$v'_x = \frac{1}{2} \cdot \frac{2x}{x^2+y^2+z^2} = \frac{x}{x^2+y^2+z^2} = \frac{x}{r^2}$$

P.S.S. ÄR $v'_y = \frac{y}{r^2}$ OCH $v'_z = \frac{z}{r^2}$

$$v''_{xx} = \frac{1\cdot(x^2+y^2+z^2) - x\cdot 2x}{(x^2+y^2+z^2)^2} = \frac{y^2+z^2-x^2}{r^4}$$

$$v''_{yy} = \frac{x^2+z^2-y^2}{r^4} \quad \text{OCH} \quad v''_{zz} = \frac{x^2+y^2-z^2}{r^4}$$

$$v''_{xx} + v''_{yy} + v''_{zz} = \frac{y^2+z^2-x^2+x^2+z^2-y^2+x^2+y^2-z^2}{r^4} =$$

$$= \frac{x^2+y^2+z^2}{r^4} = \frac{r^2}{r^4} = \frac{1}{r^2} \quad \text{VSV}$$

4a)

$$T(x,y,z) = e^{-(x^2-xy+y^2+2z^2)} \qquad \frac{dT}{d\bar{e}} = \operatorname{grad} T \cdot \frac{\bar{e}}{|\bar{e}|}$$

$$\operatorname{grad} T = ((-2x+y),\ (x-2y),\ -4z)\, e^{-(x^2-xy+y^2+2z^2)}$$

i $(1,0,1)$: $\operatorname{grad} T(1,0,1) = (-2,1,-4)\, e^{-3}$

i) $\bar{e}_1 = (-1,-2,-2) \quad |\bar{e}_1| = \sqrt{1+4+4} = 3$

$$\frac{dT}{d\bar{e}_1} = (-2,1,-4)\, e^{-3} \cdot \frac{1}{3}(-1,-2,-2) = \frac{e^{-3}}{3}\left(2-2+8\right) = \frac{8}{3} e^{-3}$$

ii) $\bar{e}_2 = (-2,-2,-1) \quad |\bar{e}_2| = 3$

$$\frac{dT}{d\bar{e}_2} = (-2,1,-4)\, e^{-3} \cdot \frac{1}{3}(-2,-2,-1) = \frac{e^{-3}}{3}(4-2+4) = 2e^{-3}$$

<u>SVAR</u>: $\frac{8}{3} e^{-3}$ I RIKTNINGEN $(-1,-2,-2)$

4b)

$$f(x,y) = x e^{-\cos y} \qquad P_0 = (1,\pi)$$

$$f(1,\pi) = e^{-\cos \pi} = e^{-(-1)} = e$$

$f'_x(x,y) = e^{-\cos y} \qquad f'_x(1,\pi) = e \qquad f''_{xx} = 0$

$f'_y(x,y) = x \sin y\, e^{-\cos y} \qquad f'_y(1,\pi) = \sin \pi\, e^{-\cos \pi} = 0 \cdot e = 0$

$f''_{xy}(x,y) = \sin y\, e^{-\cos y} \qquad f''_{xy}(1,\pi) = 0$

FORTS.

4b) FORTS.

$f''_{yy}(x,y) = x\cos y\, e^{-\cos y} + x\sin^2 y\, e^{-\cos y}$ $\quad f''_{yy}(1,\pi) = -1\cdot e = -e$

TAYLORS FORMEL! $f(x,y) = f(a,b) + (x-a) f'_x(a,b) + (y-b) f'_y(a,b) + \frac{(x-a)^2 f''_{xx}(a,b) + 2(x-a)(y-b) f''_{xy}(a,b) + (y-b)^2 f''_{yy}(a,b)}{2} + R_2$

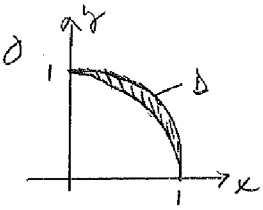
$f(x,y) = e + (x-1)e + \frac{(y-\pi)^2\cdot(-e)}{2} + R_2 =$

$= e\left(1 + x - 1 - \frac{(y-\pi)^2}{2}\right) + R_2 = e\left(x - \frac{(y-\pi)^2}{2}\right) + R_2$

SVAR! $e\cdot\left(x - \frac{(y-\pi)^2}{2}\right) + R_2$

5a)

$\iint_D x^2 y\,dx\,dy \qquad D: 1-x^2 \le y \le \sqrt{1-x^2},\ x \ge 0$



$\int_0^1 \int_{1-x^2}^{\sqrt{1-x^2}} x^2 y\,dy\,dx = \int_0^1 \left[\frac{x^2y^2}{2}\right]_{1-x^2}^{\sqrt{1-x^2}} dx =$

$\int_0^1 \frac{x^2}{2}\left[(1-x^2) - (1-x^2)^2\right] dx = \int_0^1 \frac{x^2}{2}\left(1 - x^2 - 1 + 2x^2 - x^4\right) dx =$

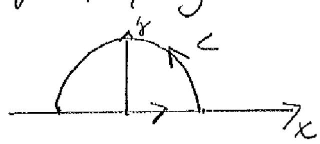
$= \int_0^1 \frac{x^2}{2}\cdot(x^2 - x^4)\,dx = \int_0^1 \left(\frac{x^4}{2} - \frac{x^6}{2}\right) dx = \left[\frac{1}{2}\left(\frac{x^5}{5} - \frac{x^7}{7}\right)\right]_0^1 =$

$= \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) = \frac{1}{2}\left(\frac{7-5}{35}\right) = \frac{1}{35}$ $\qquad$ SVAR! $\frac{1}{35}$

5b)

$\int_C (x^2+y^2)\,dx + xy\,dy \qquad C: x^2+y^2 \le 1,\ y \ge 0$

ALT LÖSN. ANVÄND GREENS FORMEL!



$C_1: x^2+y^2 = 1 \qquad -1 \le x \le 1$

$C_2: y = 0 \qquad -1 \le x \le 1$

$C_1: \begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases} \quad \begin{matrix} dx = -\sin\theta\,d\theta \\ dy = \cos\theta\,d\theta \end{matrix} \qquad \int_{C_1} = \int_0^{\pi} 1\cdot(-\sin\theta)\,d\theta + \cos\theta\sin\theta\cos\theta\,d\theta$

$= \int_0^{\pi} -\sin\theta(1-\cos^2\theta)\,d\theta = \left| \begin{matrix} \cos\theta = t \\ -\sin\theta\,d\theta = dt \\ \theta=\pi \Rightarrow t=-1 \\ \theta=0 \Rightarrow t=1 \end{matrix} \right| = \int_1^{-1} (1-t^2)\,dt = \left[t - \frac{t^3}{3}\right]_1^{-1} =$

$= -1 + \frac{1}{3} - \left(1 - \frac{1}{3}\right) = -\frac{2}{3} - \frac{2}{3} = -\frac{4}{3}$

$C_2: \begin{cases} x = t \\ y = 0 \end{cases} \quad \begin{matrix} dx = dt \\ dy = 0 \end{matrix} \quad -1 \le t \le 1 \qquad \int_{C_2} = \int_{-1}^{1} t^2\,dt = \left[\frac{t^3}{3}\right]_{-1}^{1} = \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3}$

TILLS. $\int_C = \int_{C_1} + \int_{C_2} = -\frac{2}{3}$. $\qquad$ SVAR! $-\frac{2}{3}$

6.

$$\begin{cases} x+2y+z=0 \\ 3x-y-z=0 \\ 2x-3y+cz=0 \end{cases}$$

HOMOGENT EKV. SYSTEM. $A\bar{x}=\bar{0}$
OÄNDLIGT MÅNGA LÖSNINGAR
DÅ $\det A = 0$.

$$\det A = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & -1 \\ 2 & -3 & c \end{vmatrix} = -c-4-9+2-3-6c = -7c-14$$

$\det A = 0 \iff c=-2$. SÄTT IN $c=-2$ OCH LÖS SYST.

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & -3 & -2 & 0 \end{array}\right) \begin{array}{c} (-3) \ (-2) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -7 & -4 & 0 \\ 0 & -7 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -7 & -4 & 0 \end{array}\right)$$

$-7y-4z=0 \iff y=-\frac{4}{7}z$

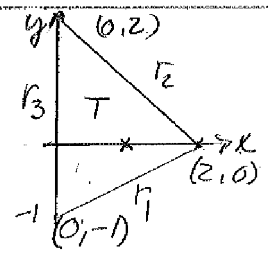
$x+2y+z=0 \iff x=-2y-z=\frac{8}{7}z-z=\frac{z}{7}$

SÄTT $z=7t \Rightarrow y=-4t$ OCH $x=t$

LINJEN ÄR: $\begin{cases} x=t \\ y=-4t \\ z=7t \end{cases}$ <u>SVAR</u>! $c=-2$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t\begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$

7.

$u(x,y) = 1-x^2+2x-y^2$



$u'_x = -2x+2 \quad u'_x=0$
$u'_y=-2y \quad u'_y=0$ $\Big\} \Rightarrow \begin{matrix} x=1 \\ y=0 \end{matrix}$

$(1,0)$ ÄR INRE STAT. PUNKT. $u(1,0) = 1-1+2=2$

RÄNDER! r_1: $y+1=\frac{1}{2}\cdot x \iff y=\frac{x}{2}-1 \quad 0\le x\le 2$

$u(x,\frac{x}{2}-1) = 1-x^2+2x-(\frac{x}{2}-1)^2 = 1-x^2+2x-\frac{x^2}{4}+x-1 = -\frac{5}{4}x^2+3x = f(x)$

$f'(x) = -\frac{5}{2}x+3$, $f'(x)=0 \iff x=6/5$ $\quad f(6/5) = -\frac{5}{4}\cdot\frac{36}{25}+3\cdot\frac{6}{5} = \frac{9}{5}$

r_2! $y-2=\frac{0-2}{2-0}(x-0) \iff y=-x+2$, $0\le x\le 2$

$u(x,-x+2) = 1-x^2+2x-(-x+2)^2 = 1-x^2+2x-(x^2-4x+4) =$
$= -2x^2+6x-3 = g(x) \quad g'(x)=-4x+6 \quad g'(x)=0 \Rightarrow x=\frac{3}{2}$

$g(\frac{3}{2}) = -2\cdot\frac{9}{4}+6\cdot\frac{3}{2}-3 = -\frac{9}{2}+9-3 = \frac{3}{2}$.

r_3! $x=0 \Rightarrow u(0,y)=1-y^2=h(y) \quad h'(y)=-2y \quad h'(y)=0$
$\iff y=0 \quad h(0)=1$.

HÖRN! $u(0,-1)=-1$, $u(2,0)=1$, $u(0,2)=-3$

<u>SVAR</u>! STÖRSTA VÄRDE $=2$, MINSTA VÄRDE $=-3$

8. $\sum_{n=1}^{\infty} \frac{n^2 x^{2n}}{5^n} = \sum_{n=1}^{\infty} \frac{n^2}{5^n} \cdot x^{2n} = \left| x^2 = y \right| = \sum_{n=1}^{\infty} \frac{n^2}{5^n} \cdot y^n = \sum_{n=1}^{\infty} a_n y^n$

SÖK KONVERGENSRADIE. $R = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$

$= \lim_{n\to\infty} \frac{\frac{n^2}{5^n}}{\frac{(n+1)^2}{5^{n+1}}} = \lim_{n\to\infty} \frac{n^2}{5^n} \cdot \frac{5^{n+1}}{(n+1)^2} = \lim_{n\to\infty} \frac{5n^2}{(n+1)^2} = 5$

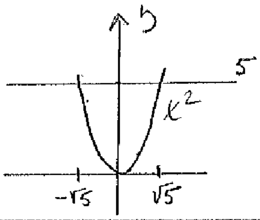
SERIEN KONV. $-5 < y < 5 \iff -5 < x^2 < 5$

$y = 5$ $\sum_{n=1}^{\infty} \frac{n^2}{5^n} \cdot 5^n = \sum_{n=1}^{\infty} n^2$ $\lim_{n\to\infty} n^2 \neq 0$!! DIV.

$y = -5$ $\sum_{n=1}^{\infty} \frac{n^2}{5^n} \cdot (-5)^n = \sum_{n=1}^{\infty} \frac{n^2}{5^n} \cdot (-1)^n \cdot 5^n = \sum_{n=1}^{\infty} (-1)^n \cdot n^2$

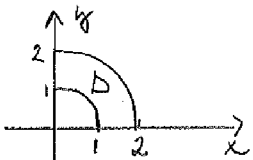
SOM ÄR DIV DÅ $\lim_{n\to\infty} n^2 \neq 0$

!! $-5 < x^2 < 5 \iff$



SVAR: $-\sqrt{5} < x < \sqrt{5}$

9. $\iint_D \frac{y\,dx\,dy}{x^2+y^2+x} =$



$D: \; 1 \le x^2+y^2 \le 4$

$x \geq 0, \; y \geq 0$

$\left| \begin{matrix} \text{POL. KOORD} \\ 0 \le \theta \le \frac{\pi}{2} \\ 1 \le r \le 2 \end{matrix} \right| = \int_0^{\frac{\pi}{2}} \int_1^2 \frac{r\sin\theta \cdot r}{r^2 + r\cos\theta} \, dr\,d\theta = \int_1^2 \int_0^{\frac{\pi}{2}} \frac{r(r\sin\theta)}{r(r+\cos\theta)} \, d\theta\,dr =$

$= \int_1^2 \int_0^{\frac{\pi}{2}} \left(\frac{r\sin\theta}{r+\cos\theta} \right) d\theta\,dr = \left| \begin{matrix} \cos\theta = t \\ -\sin\theta\,d\theta = dt \\ \theta = 0 \Rightarrow t = 1 \\ \theta = \frac{\pi}{2} \Rightarrow t = 0 \end{matrix} \right| = \int_1^2 \int_1^0 -\frac{r\,dt}{r+t} \, dr =$

$= \int_1^2 \left[r\ln(r+t) \right]_1^0 dr = \int_1^2 \left(-r\ln r + r\ln(r+1) \right) dr$

$\left| \text{PARTIELL INT} \right| = \left[-\frac{r^2}{2}\ln r \right]_1^2 - \int_1^2 -\frac{r^2}{2} \cdot \frac{1}{r}\,dr + \left[\frac{r^2}{2}\ln(r+1) \right]_1^2$

$- \int_1^2 \frac{r^2}{2} \cdot \frac{1}{r+1}\,dr = -2\ln 2 + \int_1^2 \frac{r}{2}\,dr + 2\ln 3 - \frac{1}{2}\ln 2 - \int_1^2 \frac{r^2}{2(r+1)}\,dr =$

$= 2\ln 3 - \frac{5}{2}\ln 2 + \frac{1}{2} \int_1^2 \left(r - \frac{r^2}{r+1} \right) dr = \left| \begin{matrix} r - \frac{r^2}{r+1} = \frac{r^2+r-r^2}{r+1} = \frac{r}{r+1} = \frac{r+1-1}{r+1} \\ = 1 - \frac{1}{r+1} \end{matrix} \right|$

$= 2\ln 3 - \frac{5}{2}\ln 2 + \frac{1}{2} \int_1^2 \left(1 - \frac{1}{r+1} \right) dr = 2\ln 3 - \frac{5}{2}\ln 2 + \frac{1}{2} \left[r - \ln|r+1| \right]_1^2$

FORTS.

9. FORTS FRÅN SID 7.

$$= 2\ln 3 - \frac{5}{2}\ln 2 + \frac{1}{2}(2 - \ln 3 - 1 + \ln 2) = \frac{3}{2}\ln 3 - 2\ln 2 + \frac{1}{2}.$$

SVAR: $\frac{3}{2}\ln 3 - 2\ln 2 + \frac{1}{2}$.

10.

$$f(x,y) = \begin{cases} \dfrac{x^2 - xy}{x+y} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

a) $$f'_x(x,y) = \frac{(2x-y)(x+y) - (x^2 - xy)}{(x+y)^2} = \frac{2x^2 + 2xy - xy - y^2 - x^2 + xy}{(x+y)^2}$$

$$= \frac{x^2 + 2xy - y^2}{(x+y)^2} \qquad (x,y) \neq (0,0)$$

$$f'_y(x,y) = \frac{-x(x+y) - (x^2 - xy)}{(x+y)^2} = \frac{-x^2 - xy - x^2 + xy}{(x+y)^2} =$$

$$= \frac{-2x^2}{(x+y)^2} \qquad (x,y) \neq (0,0)$$

b) $$f'_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f'_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{h^2}{h} - 0}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

$$f'_y(x,y) = \lim_{k \to 0} \frac{f(x,y+k) - f(x,y)}{k}$$

$$f'_y(0,0) = \lim_{k \to 0} \frac{f(0,k) - f(0,0)}{k} =$$

$$= \lim_{k \to 0} \frac{\frac{0}{k} - 0}{k} = 0$$

SVAR a) $f'_x = \dfrac{x^2 + 2xy - y^2}{(x+y)^2}$ $\quad f'_y = \dfrac{-2x^2}{(x+y)^2}$ $\quad (x,y) \neq (0,0)$

b) $f'_x(0,0) = 1$, $\quad f'_y(0,0) = 0$