FÖRSLAG TILL LÖSNING TENTAMEN 2006-05-19

MATEMATIK II FÖR CL, 5B1123.

1. a) ℓ_1 GÅR GENOM $A=(3,1,4)$ OCH $B=(2,1,3)$

ℓ_2 " " $C=(1,-1,2)$ OCH $D=(0,5,3)$

ℓ_1 : $\bar{v}_1 = \overrightarrow{AB} = (2,1,3)-(3,1,4) = (-1,0,-1)$

ℓ_2 : $\bar{v}_2 = \overrightarrow{CD} = (0,5,3)-(1,-1,2) = (-1,6,1)$

HITTA VINKELN MELLAN $\bar{v}_1$ OCH $\bar{v}_2$.

SKALÄRPRODUKTEN GER $\bar{v}_1 \cdot \bar{v}_2 = |\bar{v}_1| \cdot |\bar{v}_2| \cdot \cos\theta$ (*)

$\bar{v}_1 \cdot \bar{v}_2 = (-1,0,-1)\cdot(-1,6,1) = 1+0-1 = 0$

$|\bar{v}_1| = \sqrt{(-1)^2+0^2+(-1)^2} = \sqrt{2}$ $\quad |\bar{v}_2| = \sqrt{(-1)^2+6^2+1^2} = \sqrt{38} =$

INSÄTT I (*) $\quad 0 = \sqrt{2}\cdot\sqrt{38}\cos\theta \Leftrightarrow \cos\theta = 0 \Leftrightarrow \theta = \frac{\pi}{2}$

SVAR! LINJERNA ÄR VINKELRÄTA

1b) $P_0 = (1,2,-3)$ $\quad \pi_1: x-y+z-1=0$ $\quad \pi_2: 2x+y+z+1=0$

EKV. FÖR PLAN ! $\bar{n}_\pi \cdot (x-x_0, y-y_0, z-z_0) = 0$

$\bar{n}_\pi = \bar{n}_{\pi_1} \times \bar{n}_{\pi_2}$ $\quad \bar{n}_{\pi_1} = (1,-1,1)$, $\bar{n}_{\pi_2} = (2,1,1)$

$$\bar{n}_{\pi_1} \times \bar{n}_{\pi_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = (-1-1,\ 2-1,\ 1+2) = (-2,1,3)$$

SÖKTA PLANET ! $(-2,1,3)(x-1, y-2, z+3) = 0$

$-2(x-1)+y-2+3(z+3)=0 \Leftrightarrow -2x+2+y-2+3z+9=0$

SVAR ! $2x-y-3z-9=0$

2a) $(2,x,1)$, $(1,0,1)$, $(0,1,3)$

LÖS EKV. SYST ! $a(2,x,1)+b(1,0,1)+c(0,1,3)=\bar{0}$

RÄCKER ATT UNDERSÖKA SYSTEMETS DETERMINANT.

$$\det A = \begin{vmatrix} 2 & 1 & 0 \\ x & 0 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 0+1+0-0-2-3x = -1-3x$$

FÖR ATT VEKTORERNA SKALL VARA LIN. OBEROENDE

MÅSTE ENDA LÖSNINGEN VARA $a=b=c=0$

HOMOGENT EKV. SYSTEM HAR TRIV. LÖSNING DÅ

$\det A \neq 0 \Rightarrow -1-3x \neq 0 \Leftrightarrow x \neq -\frac{1}{3}$ $\quad$ SVAR! $x \neq -\frac{1}{3}$

2b)
$$A = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix} \quad \bar{v}_1 = (1,0,2) \quad \bar{v}_2 = (0,2,-5) \quad \bar{v}_3 = (0,1,-3)$$

$A\bar{v} = \lambda \bar{v}$ λ ÄR EGENVÄRDE $(\bar{v} \neq 0)$

$$A\bar{v}_1 = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \therefore \lambda_1 = 3$$

$$A\bar{v}_2 = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -5 \end{pmatrix} \quad \therefore \lambda_2 = 1$$

$$A\bar{v}_3 = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -0 \end{pmatrix} = 0\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \quad \therefore \lambda_3 = 0$$

TRE REELLA OLIKA EGENVÄRDEN $\Rightarrow$ A DIAGONALISERBAR.

SVAR! EGENVÄRDENA ÄR 0, 1, 3, A ÄR DIAGONALISERBAR.

3a)
$$\sum_{n=1}^{\infty} n \sin\frac{\pi}{n} \qquad a_n = n \cdot \sin\frac{\pi}{n} = \frac{1}{\frac{1}{n}} \sin\frac{\pi}{n} = \frac{\sin\frac{\pi}{n}}{\frac{1}{n}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sin\frac{\pi}{n}}{\frac{1}{n}} = \left| \begin{array}{l} \frac{1}{n} = t \\ n\to\infty \Rightarrow t\to 0 \end{array} \right| = \lim_{t\to 0} \frac{\sin \pi t}{t}$$

$$= \lim_{t\to 0} \pi \frac{\sin \pi t}{\pi t} \quad \left| \lim_{x\to 0} \frac{\sin x}{x} = 1 \right| = \lim_{t\to 0} \pi \cdot 1 = \pi \neq 0$$

ETT NÖDV. VILLKOR FÖR KONV. ÄR $\lim_{n\to\infty} a_n = 0$

$\therefore$ $\sum_{n=1}^{\infty} n \sin\frac{\pi}{n}$ ÄR DIVERGENT.

3b)
$u(x,y) = h(x^2y)$ VISA $x^2u''_{xx} - xu'_x - 4y^2u''_{yy} = 0$

$u'_x = 2xy\, h'(x^2y)$ $\quad u''_{xx} = 2y\, h'(x^2y) + 2xy \cdot 2xy\, h''(x^2y)$

$u'_y = x^2 h'(x^2y)$ $\quad u''_{yy} = x^2 \cdot x^2\, h''(x^2y)$

$VL = x^2u''_{xx} - xu'_x - 4y^2u''_{yy} = x^2(2y\,h' + 4x^2y^2h'')$

$- x(2xy\,h') - 4y^2(x^4h'') = 2x^2y\,h' + 4x^4y^2h'' - 2x^2y\,h' - 4x^4y^2h''$

$= 0 = HL$ VSV.

4a) $x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y}$ $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $r = \sqrt{x^2+y^2}$, $\theta = \arctan\frac{y}{x}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} = \frac{\partial f}{\partial r}\cdot\frac{x}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial \theta}\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot -\frac{y}{x^2}$$

$$= \frac{x}{r}\frac{\partial f}{\partial r} - \frac{y}{x^2}\cdot\frac{1}{\frac{x^2+y^2}{x^2}}\cdot\frac{\partial f}{\partial \theta} = \frac{x}{r}\cdot\frac{\partial f}{\partial r} - \frac{y}{x^2+y^2}\frac{\partial f}{\partial \theta} =$$

$$= \frac{x}{r}\frac{\partial f}{\partial r} - \frac{y}{r^2}\frac{\partial f}{\partial \theta}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial f}{\partial \theta}\cdot\frac{\partial \theta}{\partial y} = \frac{\partial f}{\partial r}\cdot\frac{y}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial \theta}\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x}$$

$$= \frac{y}{r}\frac{\partial f}{\partial r} + \frac{1}{x}\cdot\frac{1}{\frac{x^2+y^2}{x^2}}\frac{\partial f}{\partial \theta} = \frac{y}{r}\frac{\partial f}{\partial r} + \frac{x}{x^2+y^2}\frac{\partial f}{\partial \theta} =$$

$$= \frac{y}{r}\frac{\partial f}{\partial r} + \frac{x}{r^2}\frac{\partial f}{\partial \theta}.$$

$$x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = \frac{x^2}{r}\frac{\partial f}{\partial r} - \frac{xy}{r^2}\frac{\partial f}{\partial \theta} - \frac{y^2}{r}\frac{\partial f}{\partial r} - \frac{xy}{r^2}\frac{\partial f}{\partial \theta} =$$

$$= \frac{x^2-y^2}{r}\frac{\partial f}{\partial r} - \frac{2xy}{r^2}\frac{\partial f}{\partial \theta} = \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r}\frac{\partial f}{\partial r} - \frac{2r^2\cos\theta\sin\theta}{r^2}\cdot\frac{\partial f}{\partial \theta} =$$

$$= r\cos 2\theta\frac{\partial f}{\partial r} - \sin 2\theta\frac{\partial f}{\partial \theta}.$$

<u>SVAR</u>: $r\cos 2\theta\frac{\partial f}{\partial r} - \sin 2\theta\frac{\partial f}{\partial \theta}$

4b) $f(x,y) = xy + \frac{8}{x} + \frac{1}{y}$ $x \neq 0,\ y \neq 0$

$f'_x = y - \frac{8}{x^2}$ $f'_x = 0 \Rightarrow y - \frac{8}{x^2} = 0$ I

$f'_y = x - \frac{1}{y^2}$ $f'_y = 0 \Rightarrow x - \frac{1}{y^2} = 0$ II

I: $y = \frac{8}{x^2}$ INSATT I II GER $x - \frac{x^4}{64} = 0$

$x\left(1 - \frac{x^3}{64}\right) = 0 \Leftrightarrow x = 0$, $x^3 = 64 \Leftrightarrow x = 4$.

$x = 4$ GER $y = \frac{1}{2}$. ∴ STAT. PUNKT I $(4, \frac{1}{2})$

$f''_{xx} = +\frac{16}{x^3}$ $f''_{xy} = 1$ $f''_{yy} = \frac{2}{y^3}$

$f''_{xx}(4,\frac{1}{2}) = \frac{16}{64} = \frac{1}{4} = A$ $f''_{yy}(4,\frac{1}{2}) = \frac{2}{\frac{1}{8}} = 16 = C$ $f''_{xy} = 1 = B$

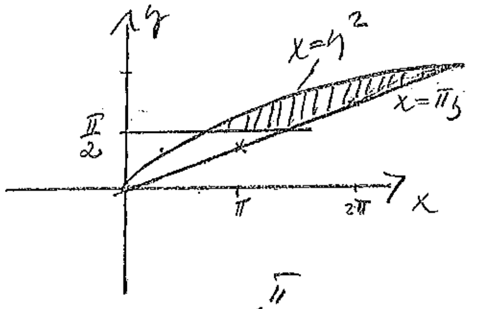
$AC - B^2 = \frac{1}{4}\cdot 16 - 1 = 3 > 0$ $f''_{xx}(4,\frac{1}{2}) > 0$ ∴ LOK MIN.

<u>SVAR</u>: $f(4,\frac{1}{2}) = 6$ ÄR LOKAL MINPUNKT.

5a) $\iint_D \cos\frac{x}{y}\,dx\,dy \qquad D=\{(x,y): y^2\le x\le \pi y,\ y\ge \frac{\pi}{2}\}$

$$= \int_{\frac{\pi}{2}}^{\pi}\left(\int_{y^2}^{\pi y}\cos\frac{x}{y}\,dx\right)dy =$$

$y^2=\pi y$
$y^2-\pi y=0$
$y(y-\pi)=0$
$y=0,\ y=\pi$



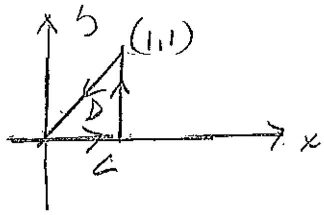
$$=\int_{\frac{\pi}{2}}^{\pi}\left[y\sin\frac{x}{y}\right]_{y^2}^{\pi y}dy = \int_{\frac{\pi}{2}}^{\pi}(y\sin\pi - y\sin y)\,dy = \int_{\frac{\pi}{2}}^{\pi} -y\sin y\,dy$$

$$= \left|\begin{array}{l}\text{part.}\\ \text{int}\end{array}\right| = \left[y\cos y\right]_{\frac{\pi}{2}}^{\pi} - \int_{\frac{\pi}{2}}^{\pi}\cos y\,dy = \left[y\cos y - \sin y\right]_{\frac{\pi}{2}}^{\pi} =$$

$$= \pi\cdot(-1) - 0 - (0-1) = 1-\pi.$$

SVAR: $1-\pi$

5b) $\int_C y^2\,dx + x^2\,dy$



SLUTET OMRÅDE. INGA SING. PUNKTER. GREENS FORMEL KAN ANVÄNDAS.

$P=y^2 \quad Q=x^2 \quad \frac{\partial P}{\partial y}=2y \quad \frac{\partial Q}{\partial x}=2x$

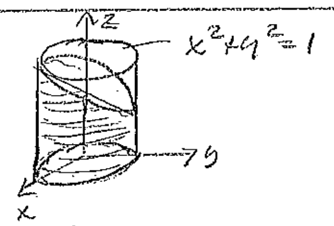
$$\int_C = \iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy = \int_0^1\int_y^1(2x-2y)\,dx\,dy = \int_0^1\left[x^2-2xy\right]_y^1 =$$

$$=\int_0^1\left(1-2y-(y^2-2y^2)\right)dy = \int_0^1(1-2y+y^2)\,dy = \left[y-y^2+\frac{y^3}{3}\right]_0^1$$

$$= 1-1+\frac{1}{3}=\frac{1}{3}.$$ (ALT. RÄKNA LÄNGS RÄNDERNA)

SVAR: $\frac{1}{3}$

6. $x^2+y^2=1 \qquad z=x+2y+3,\ z=0$



$$V=\iint_{x^2+y^2\le1}\left(\int_0^{x+2y+3}dz\right)dx\,dy = \iint_{x^2+y^2\le 1}(x+2y+3)\,dx\,dy =$$

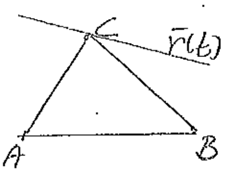
$$= \left|\begin{array}{l}\text{POL. KOORD}\\ dx\,dy = r\,dr\,d\theta\\ 0\le r\le 1\\ 0\le\theta\le 2\pi\end{array}\right| = \int_0^{2\pi}\int_0^1(r\cos\theta+2r\sin\theta+3)\,r\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^3}{3}\cos\theta+\frac{2r^3}{3}\sin\theta+\frac{3r^2}{2}\right]_0^1$$

$$=\int_0^{2\pi}\left(\frac{1}{3}\cos\theta+\frac{2}{3}\sin\theta+\frac{3}{2}\right)d\theta = \left[\frac{1}{3}\sin\theta-\frac{2}{3}\cos\theta+\frac{3\theta}{2}\right]_0^{2\pi} =$$

$$=0-\frac{2}{3}+3\pi-\left(-\frac{2}{3}\right)=3\pi$$

SVAR: 3π v.e

7. $A=(1,2,3)$ $B=(3,6,7)$ $\bar{r}(t)=(2,3-t,3+t)$



$$A=\frac{1}{2}\left|\vec{AC}\times\vec{AB}\right| \qquad A=3.$$

$$\vec{AC}=(2,3-t,3+t)-(1,2,3)=(1,1-t,t)$$
$$\vec{AB}=(3,6,7)-(1,2,3)=(2,4,4)$$

$$\vec{AC}\times\vec{AB}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 1-t & t\\ 2 & 4 & 4\end{vmatrix}=(4(1-t)-4t\ ,\ 2t-4\ ,\ 4-2(1-t))=$$

$$=(4-8t\ ,\ 2t-4,\ 2+2t)=2(2-4t,\ t-2,\ 1+t)$$

$$|\vec{AC}\times\vec{AB}|=2\sqrt{(2-4t)^2+(t-2)^2+(1+t)^2}=$$

$$=2\sqrt{4-16t+16t^2+t^2-4t+4+1+2t+t^2}=2\sqrt{18t^2-18t+9}$$

$$A=3 \Rightarrow \frac{1}{2}\cdot 2\sqrt{18t^2-18t+9}=3 \Leftrightarrow 18t^2-18t+9=9$$

$$18t^2-18t=0 \Leftrightarrow 18t(t-1)=0 \quad \therefore\ t=0\ ,\ t=1$$

$t=0$ GER $C=(2,3,3)$, $t=3$ GER $C=(2,2,4)$

<u>SVAR!</u> $C=(2,3,3)$ ELLER $C=(2,2,4)$

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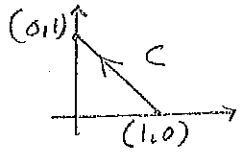
8. $$\int_C \left(\frac{y}{\cos^2 xy}+\sin x\right)dx+\left(\frac{x}{\cos^2(xy)}-\sin y\right)dy$$

$$C:\ \begin{cases} x=\cos^2 t\\ y=\sin^2 t\end{cases} \qquad 0\le t\le \frac{\pi}{2}$$
$$x+y=1$$



$$P=\frac{y}{\cos^2(xy)}+\sin x \qquad \frac{\partial P}{\partial y}=\frac{1\cdot\cos^2(xy)-y\cdot 2\cos(xy)(-\sin(xy))\cdot x}{\cos^4(xy)}=\frac{\cos xy+2xy\sin xy}{\cos^3(xy)}$$

$$Q=\frac{x}{\cos^2(xy)}-\sin y \qquad \frac{\partial Q}{\partial x}=\frac{1\cdot\cos^2(xy)-x\cdot 2\cos(xy)(-\sin(xy))\cdot y}{\cos^4(xy)}=\frac{\cos xy+2xy\sin xy}{\cos^3(xy)}$$

$\therefore\ \dfrac{\partial P}{\partial y}=\dfrac{\partial Q}{\partial x}$ VI KAN BYTA VÄG.

SINGULÄRA PUNKTER: $\cos(xy)=0 \Leftrightarrow xy=\frac{\pi}{2}+n\cdot\pi$

PUNKTERNA LIGGER I OMRÅDET $\{(x,y): |xy|\ge\frac{\pi}{2}\}$. I DET AKTUELLA OMRÅDET ÄR $|x|\le 1$, $|y|\le 1$, DVS $|xy|\le 1<\frac{\pi}{2}$ $\therefore$ INGA SINGULÄRA PUNKTER I OMRÅDET. VÄLJ t.ex VÄGEN LÄNGS AXLARNA SOM NY VÄG. (FORTS)

8 FORTS

LÄNGS X-AX: $C_1: \begin{cases} x = t & dx = dt \\ y = 0 & dy = 0 \end{cases}$ $\int_{C_1} = \int_1^0 \sin t \, dt =$

$= \left[-\cos t\right]_1^0 = -\cos 0 + \cos 1 = -1 + \cos 1$

LÄNGS y-AX: $C_2: \begin{cases} x = 0 & dx = 0 \\ y = t & dy = dt \end{cases}$ $\int_{C_2} = \int_0^1 -\sin t \, dt =$

$= \left[\cos t\right]_0^1 = \cos 1 - \cos 0 = \cos 1 - 1$

$\int_C = \int_{C_1} + \int_{C_2} = -1 + \cos 1 + \cos 1 - 1 = 2(\cos 1 - 1).$

<u>SVAR</u>: $2(\cos 1 - 1)$

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9. a) $3x^2 - 8xy + 9y^2 = 1$. SKRIV KURVAN PÅ HUVUDAXELFORM. 1) BESTÄM EGENVÄRDEN TILL A

DÅ $A = \begin{pmatrix} 3 & -4 \\ -4 & 9 \end{pmatrix}$ $\begin{vmatrix} 3-\lambda & -4 \\ -4 & 9-\lambda \end{vmatrix} = (3-\lambda)(9-\lambda) - 16 =$

$= 27 - 3\lambda - 9\lambda + \lambda^2 - 16 = \lambda^2 - 12\lambda + 11 = 0.$

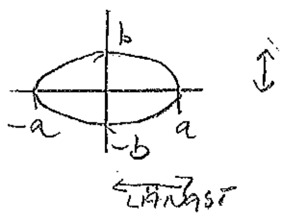
$\lambda = 6 \pm \sqrt{36-11}$ $\lambda_1 = 6+5 = 11$ $\lambda_2 = 6-5 = 1$

$\Rightarrow 3x^2 - 8xy + 9y^2 = 11u^2 + v^2$

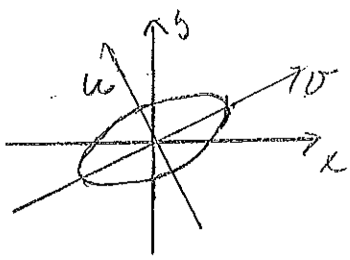
$\therefore 11u^2 + v^2 = 1 \Leftrightarrow \frac{u^2}{\left(\frac{1}{\sqrt{11}}\right)^2} + v^2 = 1$ $\therefore$ EN ELLIPS.

AVSTÅND TILL ORIGO: MINST ÄR $\frac{1}{\sqrt{11}}$

STÖRST ÄR 1.

(ALLMÄNT $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  KORTAST)

DENNA ELLIPS SER UT UNGEFÄR SÅ HÄR



b) AREAN I EN ELLIPS ÄR $\pi \cdot a \cdot b \Rightarrow \pi \cdot \frac{1}{\sqrt{11}} \cdot 1 = \frac{\pi}{\sqrt{11}}$

AU. → FORTS.

9b) ALT. 1: ANV. SUB.: $\displaystyle\iint_{11u^2+v^2\le 1} du\,dv = \left| \begin{array}{ll} u = \frac{1}{\sqrt{11}} r\cos\theta & du\,dv = \frac{r}{\sqrt{11}}\,dr\,d\theta \\ v = r\sin\theta & 0\le r\le 1 \\ & 0\le\theta\le 2\pi \end{array} \right|$

$$= \int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{11}}\, r\,dr\,d\theta = \frac{\pi}{\sqrt{11}}.$$

ALT. 2: TAG FRAM FORMELN FÖR AREAN AV EN GODTYCKLIG ELLIPS. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (SE ÄVEN ENVARIABEL-BOKEN S.312)

$y^2 = b^2 - \frac{b^2}{a^2}x^2 \qquad y = \pm\sqrt{b^2-\frac{b^2}{a^2}x^2}, \quad -a\le x\le a$

$$A = \int_{-a}^{a}\int_{-\sqrt{b^2-\frac{b^2}{a^2}x^2}}^{\sqrt{b^2-\frac{b^2}{a^2}x^2}} dy\,dx = \int_{-a}^{a}\Big[y\Big]_{-\sqrt{b^2-\frac{b^2}{a^2}x^2}}^{\sqrt{b^2-\frac{b^2}{a^2}x^2}} dx =$$

$$= \int_{-a}^{a} 2\sqrt{b^2-\frac{b^2}{a^2}x^2}\,dx = \int_{-a}^{a} 2b\sqrt{1-\frac{x^2}{a^2}}\,dx \left| \begin{array}{l} \frac{x}{a} = \sin t \\ dx = a\cos t\,dt \\ x=a \Rightarrow t=\frac{\pi}{2} \\ x=-a \Rightarrow t=-\frac{\pi}{2} \end{array} \right| =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2b\sqrt{1-\sin^2 t}\cdot a\cos t\,dt = 2b\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^2 t}\cdot a\cos t\,dt =$$

$$= 2ab\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt = 2ab\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 2t+1}{2}\,dt = ab\left[\frac{\sin 2t}{2}+t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= ab\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right) = ab\pi.$$

SVAR a) 1 OCH $\frac{1}{\sqrt{11}}$ l.e

b) $\frac{\pi}{\sqrt{11}}$ a.e.

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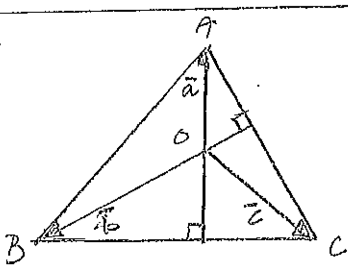
10)



LÅT HÖJDERNA FRÅN A OCH B SKÄRA VARANDRA I O.

SÄTT $\overrightarrow{OA} = \bar{a}$, $\overrightarrow{OB} = \bar{b}$ OCH $\overrightarrow{OC} = \bar{c}$

DÅ ÄR $\overrightarrow{BC} = \bar{c}-\bar{b}$, $\overrightarrow{CA} = \bar{a}-c$

OCH $\overrightarrow{AB} = \bar{b}-\bar{a}$.

$\overrightarrow{OA}$ ÄR VINKELRÄT MOT $\overrightarrow{BC} \Rightarrow \bar{a}\cdot(\bar{c}-\bar{b})=0$

$\Leftrightarrow \bar{a}\cdot\bar{c} - \bar{a}\cdot\bar{b} = 0 \Leftrightarrow \bar{a}\cdot\bar{c} = \bar{a}\cdot\bar{b}$

$\overrightarrow{OB}$ ÄR VINKELRÄT MOT $\overrightarrow{CA} \Rightarrow \bar{b}\cdot(\bar{a}-\bar{c})=0$

$\Leftrightarrow \bar{b}\cdot\bar{a} - \bar{b}\cdot\bar{c} = 0 \Leftrightarrow \bar{b}\cdot\bar{a} = \bar{b}\cdot\bar{c}$

$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a} \Rightarrow \bar{b}\cdot\bar{c} = \bar{a}\cdot\bar{c} \Leftrightarrow$

$\bar{b}\cdot\bar{c} - \bar{a}\cdot\bar{c} = 0 \Leftrightarrow \bar{c}\cdot(\bar{b}-\bar{a}) = 0 = \overrightarrow{OC}\cdot\overrightarrow{AB}$

$\therefore$ $\overrightarrow{OC}$ ÄR VINKELRÄT MOT $\overrightarrow{AB}$ V.S.V.