3 $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 2 & 4 \end{bmatrix}$, $b = \begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix}$

Minsta kvadrat: lös $A^t A x = A^t b$

dvs $\begin{bmatrix} 6 & 12 \\ 12 & 26 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 16 \\ 28 \end{pmatrix} \Rightarrow x = (20/3, -2)^t$

4 Låt $B = 6A$. $P_B(\lambda) = -(\lambda^3 - 8\lambda^2 + 18\lambda - 36)$

$P_B(\lambda) = 0$ har lösningarna $\lambda_1' = 6$, $\lambda_2' = 1 + \sqrt{-5}$

$\lambda_3' = 1 - \sqrt{-5}$

$\therefore$ $P_A(\lambda) = 0$ har lösningarna $\lambda_1 = 1$, $\lambda_2 = \frac{1+\sqrt{-5}}{6}$

$\lambda_3 = \frac{1-\sqrt{-5}}{6}$

(Notera: $|\lambda_2| = |\lambda_3| < 1$).

$(A - I)v_1 = 0 \Rightarrow v_1 = (3\ 4\ 3)^t$.

$(A - \lambda_2 I)v_2 = 0 \Rightarrow v_2 = \left(1, \frac{-1+\sqrt{-5}}{3}, \frac{2+\sqrt{-5}}{3}\right)^t$

$(A - \lambda_3 I)v_3 = 0 \Rightarrow v_3 = \left(1, \frac{-1-\sqrt{-5}}{3}, \frac{2-\sqrt{-5}}{3}\right)^t$

$x(0) = \frac{3}{10} v_1 + \alpha_2 v_2 + \alpha_3 v_3$

$x(n) = \frac{3}{10} \cdot 1^n v_1 + \alpha_2 \lambda_2^n v_2 + \alpha_3 \lambda_3^n \cdot v_3$

$\longrightarrow \frac{3}{10} v_1$ då $n \to \infty$.

5 $[L]_{\mathcal{E}} = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 5 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

$$T = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow T^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[L]_{\mathcal{B}} = T^{-1}[L]_{\mathcal{E}}T = \begin{bmatrix} 3 & 0 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

6 Ortogonal bas ges av $1, t, t^2 - 1/3$.

$$P_W(t^3) = \frac{\langle 1, t^3\rangle}{\langle 1, 1\rangle}\cdot 1 + \frac{\langle t, t^3\rangle}{\langle t, t\rangle}\cdot t + \frac{\langle t^2 - 1/3, t^3\rangle}{\|t^2 - 1/3\|^2}\cdot (t^2 - 1/3)$$

$$= 0 + \frac{\int_{-1}^{1} t^4\,dt}{\int_{-1}^{1} t^2\,dt}\cdot t + 0$$

$$= \frac{3}{5}\cdot t$$

$$\therefore P_W(2t^3 + 3t^2 + 5t + 7) = P_W(2t^3) + P_W(3t^2 + 5t + 7)$$

$$= 2\cdot\frac{3}{5}\cdot t + 3t^2 + 5t + 7$$

$$= 3t^2 + \frac{31}{5}t + 7$$