FÖRSLAG TILL LÖSNING, MATEMATIK I, DEL B FÖR CL.
2009-02-02, SF1623.

1. $$\lim_{n\to\infty}\left(1+\frac{2}{n^2}\right)^{3n^2} = \left| \frac{2}{n^2}=\frac{1}{t} \Leftrightarrow n^2 = 2t,\quad n\to\infty \Rightarrow t\to\infty \right|$$

$$= \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{3\cdot 2t} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{6t} = \lim_{t\to\infty}\left[\left(1+\frac{1}{t}\right)^{t}\right]^6 =$$

$$= \left| \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t = e \right| = e^6.$$

SVAR: e^6

2. $$\int_0^{\frac{\pi}{2}} \sin 2x \cdot \sin^3 x \, dx = \int_0^{\frac{\pi}{2}} 2\sin x \cos x \cdot \sin^3 x \, dx =$$

$$= \int_0^{\frac{\pi}{2}} 2\sin^4 x \cos x \, dx = \left| \begin{array}{l} \sin x = t \\ \cos x \, dx = dt \\ x=0 \Rightarrow t=0 \\ x=\frac{\pi}{2} \Rightarrow t=1 \end{array} \right| = \int_0^1 2t^4 \, dt =$$

$$= \left[\frac{2t^5}{5}\right]_0^1 = \frac{2}{5}$$

SVAR: $\frac{2}{5}$

3. $$y'' + 2y' - 3y = e^{-3x}$$

HOMOGEN LÖSNING:

KAR. EKV.: $r^2+2r-3=0$

$$r = -1 \pm \sqrt{1+3}$$

$$r = -1 \pm 2$$

$$r = 1, \quad r = -3$$

$$y_h = Ae^x + Be^{-3x}.$$ RESONANS!

PART. LÖSNING: $y_p = axe^{-3x}$

$$y_p' = ae^{-3x} - 3axe^{-3x} = (a-3ax)e^{-3x}$$

$$y_p'' = -3ae^{-3x} - 3(a-3ax)e^{-3x} = -6ae^{-3x} + 9axe^{-3x}$$

INSÄTTNING GER: $(9ax-6a)e^{-3x} + 2(a-3ax)e^{-3x} - 3axe^{-3x}$

$$\equiv e^{-3x} \Leftrightarrow (9ax-6a+2a-6ax-3ax)e^{-3x} \equiv e^{-3x}$$

$$-4a\,e^{-3x} \equiv e^{-3x}$$

$$-4a = 1 \Leftrightarrow a = -\frac{1}{4}$$

$$y_p = -\frac{x}{4}e^{-3x},$$

SVAR: $y = Ae^x + \left(B - \frac{x}{4}\right)e^{-3x}.$

4. $x(1+\ln x) = y + \ln y^2 \qquad (x_1, y_1) = (1,1)$

$x + x\ln x = y + 2\ln y$

IMPLICIT DERIVERING GER:

$1 + 1\cdot\ln x + x\cdot\frac{1}{x} = y' + 2\cdot\frac{y'}{y}$

INSÄTTNING AV $(1,1)$:

$1 + \ln 1 + 1 = y'(1) + 2\cdot\frac{y'(1)}{1}$

$1 + 0 + 1 = 3y'(1) \Leftrightarrow y'(1) = \frac{2}{3} = k_T$

TANGENTENS EKV: $y - y_1 = k_T(x - x_1)$

$y - 1 = \frac{2}{3}(x-1)$

$y = \frac{2}{3}x - \frac{2}{3} + 1 = \frac{2}{3}x + \frac{1}{3}$ SVAR: $y = \frac{2}{3}x + \frac{1}{3}$

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5. $$\int_1^2 \frac{dx}{\sqrt{3+2x-x^2}} = \int_1^2 \frac{dx}{\sqrt{-(x^2-2x-3)}} = \int_1^2 \frac{dx}{\sqrt{-[(x-1)^2-1-3]}} =$$

$$= \int_1^2 \frac{dx}{\sqrt{4-(x-1)^2}} = \left| \begin{matrix} x-1 = 2t \\ dx = 2dt \\ x=1 \Rightarrow t=0 \\ x=2 \Rightarrow t=\frac{1}{2} \end{matrix} \right| = \int_0^{\frac{1}{2}} \frac{2\,dt}{\sqrt{4-4t^2}} =$$

$$= \int_0^{\frac{1}{2}} \frac{2\,dt}{2\sqrt{1-t^2}} = \Big[\arcsin t\Big]_0^{\frac{1}{2}} = \arcsin\frac{1}{2} - \arcsin 0 =$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}.$$

SVAR: $\frac{\pi}{6}$

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6. $P_0 = (2,1,1) \qquad \pi:\ x + y - z + a = 0 \qquad d = \sqrt{3}$

$$d = \frac{|1\cdot 2 + 1\cdot 1 - 1\cdot 1 + a|}{\sqrt{1^2+1^2+(-1)^2}} = \frac{|2+a|}{\sqrt{3}}$$

$\frac{|2+a|}{\sqrt{3}} = \sqrt{3} \Leftrightarrow |2+a| = 3 \Leftrightarrow 2+a = \pm 3$

$2 + a = 3 \Leftrightarrow a = 1$

$2 + a = -3 \Leftrightarrow a = -5$

SVAR: $a = 1$ ELLER $a = -5$

7. $f(x) = \sqrt{x}\, e^{-\frac{\sqrt{x}}{2}}$, $x \geq 0$

$f'(x) = \frac{1}{2\sqrt{x}} e^{-\frac{\sqrt{x}}{2}} + \sqrt{x} \cdot -\frac{1}{4\sqrt{x}} e^{-\frac{\sqrt{x}}{2}} = e^{-\frac{\sqrt{x}}{2}}\left(\frac{1}{2\sqrt{x}} - \frac{1}{4}\right)$

$f'(x) = 0 \Rightarrow e^{-\frac{\sqrt{x}}{2}}\left(\frac{1}{2\sqrt{x}} - \frac{1}{4}\right) = 0$

GER $\frac{1}{2\sqrt{x}} - \frac{1}{4} = 0 \Leftrightarrow \frac{1}{2\sqrt{x}} = \frac{1}{4} \Leftrightarrow 2\sqrt{x} = 4$
$\sqrt{x} = 2$
$x = 4.$

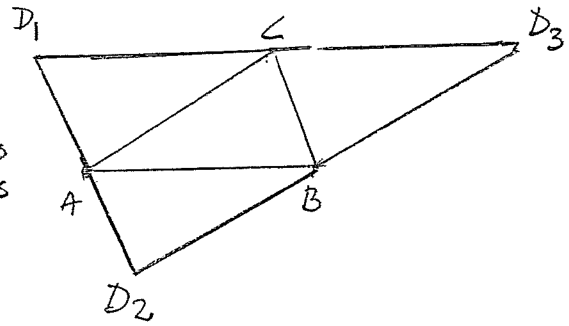
$f(4) = \sqrt{4} \cdot e^{-\frac{\sqrt{4}}{2}} = 2e^{-1} = \frac{2}{e}$

$f(0) = 0 \cdot e^{-0} = 0$

$\lim_{x \to \infty} \sqrt{x}\, e^{-\frac{\sqrt{x}}{2}} \to 0$ $\left(\lim_{x\to\infty} \frac{\sqrt{x}}{e^{\frac{\sqrt{x}}{2}}} \to 0\right.$ STANDARD GRÄNSVÄRDE$\left.\right)$

$\therefore$ $0 \leq f(x) \leq \frac{2}{e}$ SVAR! $0 \leq f(x) \leq \frac{2}{e}$

8.



EN VEKTOR FRÅN ORTEN O TILL A KALLAS $\overline{A}$ ETC.

I DE TRE OLIKA FALLEN KALLAS DET FJÄRDE HÖRNET D_1, D_2 RESP D_3

$\overline{D}_1 = \overline{C} + \overrightarrow{BA} = \overline{A} + \overrightarrow{BC}$

$\overline{D}_2 = \overline{A} + \overrightarrow{CB} = \overline{B} + \overrightarrow{CA}$

$\overline{D}_3 = \overline{B} + \overrightarrow{AC} = \overline{C} + \overrightarrow{AB}$

$\overline{A} = (0,1,0)$ $\overline{B} = (1,0,0)$, $\overline{C} = (0,0,2)$

$\overrightarrow{BA} = (0,1,0) - (1,0,0) = (-1,1,0)$

$\overrightarrow{CB} = (1,0,0) - (0,0,2) = (1,0,-2)$

$\overrightarrow{AC} = (0,0,2) - (0,1,0) = (0,-1,2)$

$\Rightarrow \overline{D}_1 = (0,0,2) + (-1,1,0) = (-1,1,2)$

$\overline{D}_2 = (0,1,0) + (1,0,-2) = (1,1,-2)$

$\overline{D}_3 = (1,0,0) + (0,-1,2) = (1,-1,2)$

SVAR! MÖJLIGA KOORD. FÖR D ÄR $(-1,1,2)$ ELLER $(1,1,-2)$ ELLER $(1,-1,2)$. AREAN ÄR 3.

I ALLA TRE FALLEN BLIR AREA FÖR $ABCD_i = 2 \cdot$ AREAN(A,B,C)

$= |\overrightarrow{AC} \times \overrightarrow{AB}| = \left\| \begin{vmatrix} e_x & e_y & e_z \\ 0 & -1 & 2 \\ 1 & -1 & 0 \end{vmatrix} \right\| = |(2,2,1)| = \sqrt{9} = 3$ $(i = 1,2,3)$

9.

$$f(x) = \begin{cases} \dfrac{x \sin x^3}{\ln(1+x^b)} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$f(x)$ ÄR KONTINUERLIG I $x=0$ DÅ $\lim_{x \to 0} f(x) = 1$

$$\lim_{x \to 0} \frac{x \sin x^3}{\ln(1+x^b)} = \lim_{x \to 0} \frac{x\left(x^3 - \frac{(x^3)^3}{3!} + O((x^3)^5)\right)}{x^b - \frac{(x^b)^2}{2} + O((x^b)^3)} =$$

$$= \lim_{x \to 0} \frac{x^4\left(1 - \frac{x^9}{3!} + O(x^{15})\right)}{x^b\left(1 - \frac{x^b}{2} + O((x^b)^2)\right)}$$ DÅ $\frac{x^4}{x^b} = 1$ ANTAR

GRÄNSVÄRDET VÄRDET 1. $\therefore$ $b=4$.

<u>SVAR:</u> $b=4$ GER $f(0)=1$

10.

$$\ln x = \int_1^x \frac{dt}{t}, \quad x > 0$$

VISA $\ln xy = \ln x + \ln y$, $x > 0$, $y > 0$

LÅT y VARA FIXT. DERIVERA: $\frac{d}{dx} \ln x = \frac{d}{dx} \int_1^x \frac{dt}{t} = \frac{1}{x}$

$$\frac{d}{dx}(\ln x + \ln y) = \frac{1}{x} + 0 = \frac{1}{x}$$

$$\frac{d}{dx}(\ln xy) = (\text{kedjeregeln}) = \frac{1}{xy} \cdot y = \frac{1}{x}$$

DESSA DERIVATOR ÄR LIKA $\Rightarrow$

$$\ln x + \ln y = \ln xy + C$$

SÄTT $x=1 \Rightarrow C=0$ FÖR ALLA y.

$\therefore$ $\ln xy = \ln x + \ln y$ VSV.