1.

$f(x) = \arctan(2\tan 2x) \qquad x = \frac{\pi}{6}$

$f'(x) = \frac{1}{1+(2\tan 2x)^2} \cdot 2(1+\tan^2 2x)\cdot 2$

$f'(\frac{\pi}{6}) = \frac{1}{1+(2\tan\frac{\pi}{3})^2} \cdot 2(1+(\tan\frac{\pi}{3})^2)\cdot 2 =$

$= \frac{1}{1+(2\sqrt{3})^2} \cdot 4(1+(\sqrt{3})^2) = \frac{1}{1+12}\cdot 4(1+3) = \frac{16}{13}.$

SVAR: $\frac{16}{13}$

2.

$\int_0^{\ln 2} \frac{e^x}{\sqrt{5-e^{2x}}}\,dx = \left| \begin{array}{l} e^x = t \\ e^x dx = dt \\ x=0 \Rightarrow t=1 \\ x=\ln 2 \Rightarrow t=2 \end{array} \right| = \int_1^2 \frac{dt}{\sqrt{5-t^2}} =$

$= \int_1^2 \frac{dt}{\sqrt{5}\sqrt{1-(\frac{t}{\sqrt5})^2}} = \left| \begin{array}{l} \frac{t}{\sqrt5} = u \\ \frac{dt}{\sqrt5} = du \\ t=1 \Rightarrow u=\frac{1}{\sqrt5} \\ t=2 \Rightarrow u=\frac{2}{\sqrt5} \end{array} \right| = \int_{\frac{1}{\sqrt5}}^{\frac{2}{\sqrt5}} \frac{du}{\sqrt{1-u^2}} = \arcsin u \Big]_{\frac{1}{\sqrt5}}^{\frac{2}{\sqrt5}}$

$= \arcsin\frac{2}{\sqrt5} - \arcsin\frac{1}{\sqrt5}$

SVAR: $\arcsin\frac{2}{\sqrt5} - \arcsin\frac{1}{\sqrt5}$

3.

$P_0 = (2,1,0) \qquad l: (x,y,z) = (3,-1,2) + t(1,0,-1)$

VÄLJ TVÅ PUNKTER PÅ LINJEN t.ex. $P_1 = (3,-1,2)$

OCH $P_2 = (4,-1,1)$

PLANETS EKV: $\bar{n}\cdot(\bar{x}-\bar{x}_0) = 0$

$\bar{n} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}$

$\overrightarrow{P_0P_1} = (3,-1,2)-(2,1,0) = (1,-2,2)$

$\overrightarrow{P_0P_2} = (4,-1,1)-(2,1,0) = (2,-2,1)$

$\bar{n} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 1 & -2 & 2 \\ 2 & -2 & 1 \end{vmatrix} = (-2+4, 4-1, -2+4) = (2,3,2)$

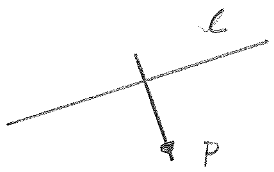
PLANET π: $(2,3,2)((x,y,z)-(2,1,0)) = 0$

$(2,3,2)(x-2, y-1, z) = 0$

$2(x-2)+3(y-1)+2z = 0 \Leftrightarrow 2x+3y+2z-7 = 0$

SVAR: $2x+3y+2z=7$.

4. $\begin{cases} x = 1 - t \\ y = 1 + t \\ z = 1 + 3t \end{cases}$ $P = (1, -1, 1)$



ALLA PUNKTER PÅ LINJEN KAN SKRIVAS SOM

$(x, y, z) = (1-t, 1+t, 1+3t)$

VEKTORN MELLAN DESSA PUNKTER OCH P ÄR

$\bar{v} = (1-t, 1+t, 1+3t) - (1,-1,1) = (-t, 2+t, 3t)$

KORTASTE AVSTÅNDET UPPSTÅR DÅ $\bar{v}$ OCH l ÄR VINKELRÄTA.

DVS $(-1,1,3)\cdot(-t, 2+t, 3t) = 0 \iff t + 2 + t + 9t = 0$

$\iff t = -\frac{2}{11}$.

$\therefore$ PUNKTEN PÅ LINJEN ÄR $(1+\frac{2}{11}, 1-\frac{2}{11}, 1+3(\frac{-2}{11})) =$

$= (\frac{13}{11}, \frac{9}{11}, \frac{5}{11}) = \frac{1}{11}(13, 9, 5)$ SVAR! $\frac{1}{11}(13, 9, 5)$

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5. $\int_1^{\sqrt{2}} \frac{x^4+1}{x^3+2x}\,dx = \int_1^{\sqrt{2}} (x - \frac{2x^2-1}{x^3+2x})\,dx = \left|\begin{array}{l}\text{PARTIAL-}\\\text{BRÅKSUPPD}\end{array}\right| = I$

$\frac{2x^2-1}{x^3+2x} = \frac{2x^2-1}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2} = \frac{A(x^2+2) + x(Bx+C)}{x(x^2+2)}$

$= \frac{Ax^2 + 2A + Bx^2 + Cx}{x(x^2+2)}$

$x^2: \quad A + B = 2 \quad B = 2 - A = \frac{5}{2}$

$x: \quad C = 0$

$x^0: \quad 2A = -1 \Rightarrow A = -\frac{1}{2}$

$I = \int_1^{\sqrt{2}} x - (-\frac{1}{2x} + \frac{5}{2}\cdot\frac{x}{x^2+2})\,dx = \left[\frac{x^2}{2} + \frac{1}{2}\ln|x| - \frac{5}{2}\cdot\frac{1}{2}\ln|x^2+2|\right]_1^{\sqrt{2}}$

$= 1 + \frac{1}{2}\ln\sqrt{2} - \frac{5}{4}\ln 4 - (\frac{1}{2} - \frac{5}{4}\ln 3) = \frac{1}{2} + \frac{1}{2}\ln 2^{\frac{1}{2}} -$

$- \frac{5}{4}\ln 2^2 + \frac{5}{4}\ln 3 = \frac{1}{2} + \frac{1}{4}\ln 2 - \frac{10}{4}\ln 2 + \frac{5}{4}\ln 3 =$

$= \frac{1}{2} - \frac{9}{4}\ln 2 + \frac{5}{4}\ln 3.$

SVAR! $\frac{1}{2} - \frac{9}{4}\ln 2 + \frac{5}{4}\ln 3$

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6. (*) $y'' - 4y' + 20y = 48e^{2x}$ $y(0)=0$, $y'(0)=12$

HOMOGEN LÖSN: KAR. EKV: $r^2-4r+20=0$
$r = 2 \pm \sqrt{4-20}$
$r = 2 \pm 4i$

$y_h = e^{2x}(A\cos 4x + B\sin 4x)$

PART. LÖSN:

$y_p = ae^{2x}$ $y_p' = 2ae^{2x}$ $y_p'' = 4ae^{2x}$

INSATT I (*)

$4ae^{2x} - 8ae^{2x} + 20ae^{2x} = 16ae^{2x} \equiv 48e^{2x}$

$\Rightarrow a = 3.$

$y = 3e^{2x} + e^{2x}(A\cos 4x + B\sin 4x) = e^{2x}(3 + A\cos 4x + B\sin 4x)$

$y(0)=0 \Rightarrow 0 = 3 + A \Leftrightarrow A = -3$

$y' = 2e^{2x}(3 + A\cos 4x + B\sin 4x) + e^{2x}(-4A\sin 4x + 4B\cos 4x)$

$y'(0) = 2(3+A) + 4B = |A=-3| = 4B \equiv 12 \Rightarrow B = 3$

$y = 3e^{2x}(1 - \cos 4x + \sin 4x)$

<u>SVAR:</u> $y = 3e^{2x}(1-\cos 4x + \sin 4x)$

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7. $y = \ln\sqrt{\frac{1+x}{1-x}} - 2x$, STUDERA $\frac{1+x}{1-x}$:

x		-1		1	
$1+x$	$-$	0	$+$		$+$
$1-x$	$+$		$+$	0	$-$
$\frac{1+x}{1-x}$	$-$	0	$+$	$\nexists$	$-$

$\sqrt{\frac{1+x}{1-x}} > 0 \Rightarrow -1 < x < 1$,

D_f : $-1 < x < 1$, ASYMPTOTER: $x=1, x=-1$

$y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) - 2x = \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-x) - 2x$

$y' = \frac{1}{2}\cdot\frac{1}{1+x} - \frac{1}{2}\cdot\frac{-1}{1-x} - 2 = \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x}\right) - 2 =$

$= \frac{1}{2}\left(\frac{1-x+1+x}{(1+x)(1-x)}\right) - 2 = \frac{1}{(1+x)(1-x)} - 2 = \frac{1-2(1+x)(1-x)}{(1+x)(1-x)}$

$= \frac{1-2(1-x^2)}{(1+x)(1-x)}$ $y' = 0 \Rightarrow 1 - 2(1-x^2) = 0$

$1 - 2 + 2x^2 = 0$

$x^2 = \frac{1}{2}$, $x = \pm\frac{1}{\sqrt{2}}$

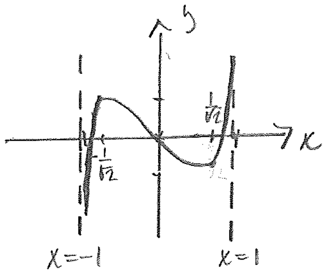
FORTS

x	-1	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
f'	∄ +	−	+	∄
f	↗	↘	↗	

∴ DÅ $x = \frac{-1}{\sqrt{2}}$ HAR f LOK. MAX

DÅ $x = \frac{1}{\sqrt{2}}$ " f LOK. MIN

$f_{max} = f\left(\frac{-1}{\sqrt{2}}\right) = \ln(\sqrt{2}-1) + \sqrt{2} \approx 0{,}5$, $f\left(\frac{1}{\sqrt{2}}\right) = -\left(\ln(\sqrt{2}-1)+\sqrt{2}\right) = -f_{max}$



$f(0) = 0$

SVAR: D_f: $-1 < x < 1$
AS: $x = -1$, $x = 1$
LOK MAX: $\left(-\frac{1}{\sqrt{2}}, \ln(\sqrt{2}-1)+\sqrt{2}\right)$
LOK MIN: $\left(\frac{1}{\sqrt{2}}, -(\ln(\sqrt{2}-1)+\sqrt{2})\right)$

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8. $y + xe^{-y} = 1$ $y(0)$: $y(0) + 0 \cdot e^{-y} = 1$ ∴ $y(0) = 1$

$\frac{d}{dx}$: $y' + e^{-y} + x \cdot -y' e^{-y} = 0$

$y'(1 - xe^{-y}) = -e^{-y}$ $y' = \frac{-e^{-y}}{1 - xe^{-y}}$ $y'(0) = -\frac{e^{-1}}{1} = -e^{-1}$

$\frac{d^2}{dx^2}$: $y''(1 - xe^{-y}) + y' \cdot (-e^{-y} - x \cdot (-y')e^{-y}) = y'e^{-y}$

$y(0) = 1$ och $y'(0) = -e^{-1}$ GER

$y''(0)(1) - e^{-1}(-e^{-1}) = -e^{-1} \cdot e^{-1}$

$y''(0) = -e^{-2} - e^{-2} = -2e^{-2}$

MACLAUR. UTV. $y(x) = 1 - \frac{x}{e} - \frac{2}{e^2} \cdot \frac{1}{2} x^2 + O(x^3) = 1 - \frac{x}{e} - \frac{x^2}{e^2} + O(x^3)$

SVAR: $y(x) = 1 - \frac{x}{e} - \frac{x^2}{e^2} + O(x^3)$

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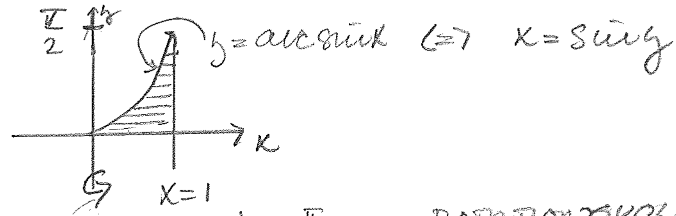
9. $y = \arcsin x$



SÖKTA VOLYMEN:

V_y = CYLINDERN MED $r=1$ OCH $h=\frac{\pi}{2}$ − ROTATIONSVOL. AV OMRÅDET MELLAN y-AXELN OCH $y=\arcsin x$.

$$V_y = \pi r^2 h - \pi\int_0^{\frac{\pi}{2}} x^2\,dy = \pi\cdot 1^2\cdot\frac{\pi}{2} - \pi\cdot\int_0^{\frac{\pi}{2}}\sin^2 y\,dy =$$

$$= \frac{\pi^2}{2} - \pi\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2y}{2}\right)dy = \frac{\pi^2}{2} - \pi\left[\frac{y}{2} - \frac{\sin 2y}{4}\right]_0^{\frac{\pi}{2}} =$$

$$= \frac{\pi^2}{2} - \pi\cdot\left(\frac{\pi}{4} - 0\right) = \frac{\pi^2}{2} - \frac{\pi^2}{4} = \frac{\pi^2}{4}.$$

<u>SVAR</u>: $\frac{\pi^2}{4}$ v.e

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10. INTEGRALKALKYLENS MEDELVÄRDESSATS: OM FUNKTIONERNA f OCH g ÄR KONTINUERLIGA OCH OM $g(x) \geq 0$ I $[a,b]$ SÅ FINNS ETT TAL ξ MELLAN a OCH b SÅ ATT

$$\int_a^b f(x)\,g(x)\,dx = f(\xi)\int_a^b g(x)\,dx \qquad \text{(sid 296, SATS 8)}$$

(SPECIALFALL $g(x)=1$ − SATS 7, SID 294)

$$\int_0^1 \frac{e^x}{1+x^2}\,dx = e^{\xi}\int_0^1\frac{dx}{1+x^2} = e^{\xi}\cdot\left[\arctan x\right]_0^1 = e^{\xi}\cdot\frac{\pi}{4}$$

STÖRSTA VÄRDET e^{ξ} ANTAR I INT. $[0,1]$ ÄR $e^1 = e$

MINSTA VÄRDET e^{ξ} ANTAR I INT. $[0,1]$ ÄR $e^0 = 1$

$$\therefore\quad 1\cdot\frac{\pi}{4} \leq \int_0^1\frac{e^x}{1+x^2}\,dx \leq e\cdot\frac{\pi}{4}$$

V.S.V.