1.

$$f(x) = \arctan\left(\frac{a^2+x^2}{a^2-x^2}\right)$$

$$f'(x) = \frac{1}{1+\left(\frac{a^2+x^2}{a^2-x^2}\right)^2} \cdot \frac{2x(a^2-x^2)-(-2x)(a^2+x^2)}{(a^2-x^2)^2} =$$

$$= \frac{1}{1+\frac{(a^2+x^2)^2}{(a^2-x^2)^2}} \cdot \frac{2xa^2-2x^3+2xa^2+2x^3}{(a^2-x^2)^2} =$$

$$= \frac{(a^2-x^2)^2}{(a^2-x^2)^2+(a^2+x^2)^2} \cdot \frac{4xa^2}{(a^2-x^2)^2} = \frac{4xa^2}{a^4-2a^2x^2+x^4+a^4+2a^2x^2+x^4}$$

$$= \frac{4xa^2}{2a^4+2x^4} = \frac{2a^2x}{a^4+x^4} \qquad \underline{\text{SVAR}}: \frac{2a^2x}{a^4+x^4}$$

2.

$$f(x) = e^{-\frac{1}{x^2}} \qquad f(1) = e^{-1}$$

$$f'(x) = -(-2x^{-3}) \cdot e^{-\frac{1}{x^2}} = \frac{2}{x^3} e^{-\frac{1}{x^2}} \qquad f'(1) = 2e^{-1}$$

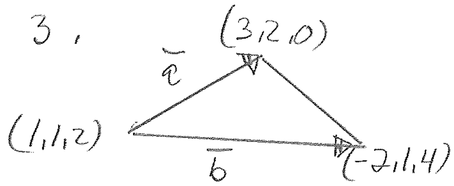
$$f''(x) = -6x^{-4} \cdot e^{-\frac{1}{x^2}} + 2x^{-3} \cdot 2 \cdot x^{-3} e^{-\frac{1}{x^2}} \qquad f''(1) = (-6+4)e^{-1} = -2e^{-1}$$

$$e^{-\frac{1}{x^2}} = e^{-1} + 2e^{-1}(x-1) - \frac{2e^{-1}}{2}(x-1)^2 + R_2(x-1)$$

$$= \frac{1}{e} + \frac{2(x-1)}{e} - \frac{(x-1)^2}{e} + R_2(x-1)$$

$$\underline{\text{SVAR}}: \ e^{-\frac{1}{x^2}} = \frac{1}{e} + \frac{2(x-1)}{e} - \frac{(x-1)^2}{e} + R_2(x-1)$$

3.



$$\bar{a} = (3,2,0)-(1,1,2) = (2,1,-2)$$

$$\bar{b} = (-2,1,4)-(1,1,2) = (-3,0,2)$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 2 & 1 & -2 \\ -3 & 0 & 2 \end{vmatrix} = (2,2,3)$$

$$A = \frac{1}{2}|\bar{a}\times\bar{b}|$$

$$|\bar{a}\times\bar{b}| = \sqrt{2^2+2^2+3^2} = \sqrt{17} \qquad A = \frac{\sqrt{17}}{2} \qquad \underline{\text{SVAR}}: \frac{\sqrt{17}}{2} \text{ a.e}$$

4.

$$f(x) = \frac{x\sqrt{x} - x}{x - x^2} \qquad x \neq 0,\ x \neq 1 \qquad D_f: x > 0$$

$$\lim_{x \to 0^+} \frac{x\sqrt{x} - x}{x - x^2} = \lim_{x \to 0^+} \frac{x(\sqrt{x} - 1)}{x(1-x)} = \lim_{x \to 0^+} \frac{\sqrt{x} - 1}{1 - x} = -1$$

$$\therefore f(0) = -1$$

$$\lim_{x \to 1} \frac{x\sqrt{x} - x}{x - x^2} = \lim_{x \to 1} \frac{\sqrt{x} - 1}{1 - x} = \lim_{x \to 1} \frac{\sqrt{x} - 1}{(1 - \sqrt{x})(1 + \sqrt{x})} =$$

$$= \lim_{x \to 1} \frac{-(1 - \sqrt{x})}{(1 - \sqrt{x})(1 + \sqrt{x})} = \lim_{x \to 1} \frac{-1}{1 + \sqrt{x}} = \frac{-1}{2} = -\frac{1}{2} \qquad \therefore f(1) = -\frac{1}{2}$$

SVAR!
$$f(x) = \begin{cases} \dfrac{x\sqrt{x} - x}{x - x^2} & x \neq 0,\ x \neq 1 \\ -1 & x = 0 \\ -\dfrac{1}{2} & x = 1 \end{cases}$$

5.

VISA $\ln x \leq x^2 - x,\ x > 0$

$$f(x) = \ln(x) - x^2 + x \qquad \text{VISA } f(x) \leq 0$$

$$f'(x) = \frac{1}{x} - 2x + 1 = \frac{1 - 2x^2 + x}{x} = \frac{-2x^2 + x + 1}{x} =$$

$$= \frac{-2(x^2 - \frac{x}{2} - \frac{1}{2})}{x} = \frac{-2(x-1)(x+\frac{1}{2})}{x}$$

$f'(x) = 0$ DÅ $x = 1$ ($x = -\frac{1}{2}$ ÄR EJ DEF.)

x	0		1	
$f'(x)$	$\nexists$	+	0	−
$f(x)$	$\nexists$	$\nearrow$	0	$\searrow$

$f(1) = 0$ ÄR ETT MAX.

$f_{max} = 0 \Rightarrow f(x) \leq 0,\ x > 0$

VSV

6.

$$\int_0^1 \frac{x\,dx}{\sqrt{2 + 2x^2 - x^4}} = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x = 0 \Rightarrow t = 0 \\ x = 1 \Rightarrow t = 1 \end{array} \right| = \int_0^1 \frac{\frac{1}{2}\,dt}{\sqrt{2 + 2t - t^2}} =$$

$$= \frac{1}{2} \int_0^1 \frac{dt}{\sqrt{3 - (t^2 - 2t + 1)}} = \frac{1}{2} \int_0^1 \frac{dt}{\sqrt{3 - (t-1)^2}} = \left| \begin{array}{l} t - 1 = \sqrt{3}u \\ dt = \sqrt{3}\,du \\ t = 0 \Rightarrow u = -\frac{1}{\sqrt{3}} \\ t = 1 \Rightarrow u = 0 \end{array} \right|$$

$$= \frac{1}{2} \int_{-\frac{1}{\sqrt{3}}}^0 \frac{\sqrt{3}\,du}{\sqrt{3 - 3u^2}} = \frac{1}{2} \int_{-\frac{1}{\sqrt{3}}}^0 \frac{du}{\sqrt{1 - u^2}} = \frac{1}{2}\Big[\arcsin u\Big]_{-\frac{1}{\sqrt{3}}}^0 = 0 - \frac{1}{2}\left(\arcsin\left(-\frac{1}{\sqrt{3}}\right)\right) =$$

6. FORTS.

$$-\frac{1}{2}\left(\arcsin\left(-\frac{1}{\sqrt{3}}\right)\right) = -\frac{1}{2}\cdot -\arcsin\frac{1}{\sqrt{3}} = \frac{1}{2}\arcsin\frac{1}{\sqrt{3}}.$$

SVAR: $\frac{1}{2}\arcsin\frac{1}{\sqrt{3}}$

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7. $P_0 = (1,-1,1)$ $\quad L: x = 2y-1 = 3z-2.$

$$L: \begin{cases} x = t \\ 2y-1 = t \iff y = \frac{1}{2}(1+t) \\ 3z-2 = t \iff z = \frac{1}{3}(2+t) \end{cases}$$

TVÅ PUNKTER PÅ LINJEN ÄR: $P_1 = (1,1,1)$ OCH $P_2 = (-5,-2,-1)$

VI SÖKER PLANETS NORMALVEKTOR $\bar{n}$.

VEKTORERNA $\overrightarrow{P_0P_1} = (1,1,1)-(1,-1,1) = (0,2,0)$ OCH $\overrightarrow{P_0P_2} = (-5,-2,-1)-(1,-1,1) = (-6,-1,-2)$ LIGGER I PLANET.

$$\bar{n} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 0 & 2 & 0 \\ -6 & -1 & -2 \end{vmatrix} = (-4, 0, 12) = -4(1,0,-3)$$

SÖKTA PLANET Π: $\bar{n}\cdot(x-1, y+1, z-1) = 0$

$\Rightarrow (1,0,-3)(x-1, y+1, z-1) = 0 \iff x-1-3(z-1) = 0$

$x-3z+2 = 0.$ SVAR: $x-3z+2=0$

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8. $$\int_0^{\pi} \frac{\sin^3 x}{4-\cos^2 x}\,dx = \int_0^{\pi} \frac{\sin^2 x\cdot\sin x}{4-\cos^2 x}\,dx = \int_0^{\pi} \frac{(1-\cos^2 x)\cdot\sin x}{4-\cos^2 x}\,dx$$

$$\left|\begin{array}{l} \cos x = t \\ -\sin x\,dx = dt \\ x=0 \Rightarrow t=1 \\ x=\pi \Rightarrow t=-1 \end{array}\right| = \int_1^{-1} \frac{-(1-t^2)}{4-t^2}\,dt = \int_{-1}^{1} \frac{1-t^2}{4-t^2}\,dt = \int_{-1}^{1} \frac{4-t^2-3}{4-t^2}\,dt$$

$$= \int_{-1}^{1}\left(1 - \frac{3}{4-t^2}\right)dt = \Big[t\Big]_{-1}^{1} - \int_{-1}^{1} \frac{3}{(2-t)(2+t)}\,dt = 1-(-1) - \int_{-1}^{1} \frac{3\,dt}{(2-t)(2+t)}$$

$$\frac{3}{(2-t)(2+t)} = \frac{A}{2-t} + \frac{B}{2+t} = \frac{2A+At+2B-Bt}{(2-t)(2+t)}$$

$A-B=0 \iff A=B$
$2A+2B=3$
$4A=3 \quad A=B=\frac{3}{4}$

FORTS

8 FORTS $I = +2 - \frac{3}{4}\int_{-1}^{1}\left(\frac{1}{2-t} + \frac{1}{2+t}\right)dt = 2 - \frac{3}{4}\Big[-\ln|2-t| + \ln|2+t|\Big]_{-1}^{1}$

$= 2 - \frac{3}{4}\left(-\ln 1 + \ln 3 - (-\ln 3 + \ln 1)\right) = 2 - \frac{3}{4}\cdot 2\ln 3 = 2 - \frac{3}{2}\ln 3$

SVAR! $2 - \frac{3}{2}\ln 3$

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9. $y = \sqrt{\frac{2}{x}}\, e^{-\frac{1}{x}}, \quad x > 0$ $\qquad \lim_{x\to\infty} \sqrt{\frac{2}{x}}\, e^{-\frac{1}{x}} \to 0$ TY $\sqrt{\frac{2}{x}} > 0$ OCH $0 < e^{-\frac{1}{x}} < 1$

$\therefore$ $y = 0$ ÄR EN ASYMPTOT.

$y' = \sqrt{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}e^{-\frac{1}{x}} + \sqrt{\frac{2}{x}}\cdot\frac{1}{x^2}e^{-\frac{1}{x}} = e^{-\frac{1}{x}}\left(-\frac{\sqrt{2}}{2x\sqrt{x}} + \frac{\sqrt{2}}{x^2\sqrt{x}}\right) =$

$= \frac{\sqrt{2}}{x\sqrt{x}}e^{-\frac{1}{x}}\left(-\frac{1}{2} + \frac{1}{x}\right) \qquad y' = 0 \Rightarrow \frac{1}{x} = \frac{1}{2} \quad \therefore x = 2$

$$\begin{array}{cccc} x & 0 & & 2 & \\ y' & & + & 0 & - \\ y & & \nearrow & & \searrow \end{array}$$

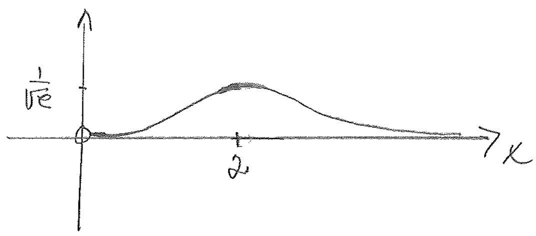
$y(2) = e^{-\frac{1}{2}}$ ÄR LOK. MAX

$\lim_{x\to 0^+} y(x) = \lim_{x\to 0^+}\sqrt{\frac{2}{x}}\cdot e^{-\frac{1}{x}} = \left|\begin{array}{c} t = \frac{1}{x} \\ t \to +\infty \end{array}\right| = \lim_{t\to\infty}\sqrt{2t}\, e^{-t} =$

$= \lim_{t\to\infty}\frac{\sqrt{2t}}{e^t} \to 0$

$\lim_{x\to 0^+} y'(x) = \lim_{x\to 0^+}\frac{\sqrt{2}}{x\sqrt{x}}e^{-\frac{1}{x}}\left(-\frac{1}{2} + \frac{1}{x}\right) = \left|\begin{array}{c} t = \frac{1}{x} \\ t \to +\infty \end{array}\right| =$

$= \lim_{t\to\infty}\frac{\sqrt{2}}{e^t}\left(-\frac{t^{\frac{3}{2}}}{2} + t^{5/2}\right) \to 0 \quad \therefore$ TANGENT I $(0,0)$



SVAR! LOK. MAX I $(2, e^{-\frac{1}{2}})$

$y = 0$ ÄR ASYMPTOT

TANGENT I $(0,0)$

10. sid 202 i PERSSON - BÖIERS (DIFF. KALKYLENS MEDELVÄRDESSATS)

ANTAG ATT f ÄR KONTINUERLIG I DET SLUTNA INTERVALLET $a \le x \le b$ OCH DERIVERBAR I DET ÖPPNA INTERVALLET $a < x < b$. DÅ FINNS MINST EN PUNKT c $a < c < b$ SÅDANT ATT $f(b) - f(a) = f'(c)(b-a)$. (*)

sid 206 i PERSSON - BÖIERS

$f'(c) > 0$ OCH $b > a \Rightarrow b - a > 0$ SÄTTS IN I (*)

$\Rightarrow f(b) - f(a) > 0$ DVS $f(a) < f(b)$

OCH VI HAR VISAT ATT OM $a < b \Rightarrow f(a) < f(b)$

DVS f ÄR EN VÄXANDE FUNKTION.

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