1.
$$f(x) = xe^{-2x}, \quad 0 \le x \le 1 \qquad f(0) = 0 \qquad f(1) = e^{-2}$$

$$f'(x) = e^{-2x} + x\cdot(-2)e^{-2x} = (1-2x)e^{-2x}$$

$$f'(x) = 0 \Rightarrow (1-2x)e^{-2x} = 0 \Leftrightarrow x = \frac{1}{2}$$

$$f(\tfrac{1}{2}) = \frac{1}{2}e^{-1}$$

$$0 < e^{-2} < \frac{1}{2}e^{-1}$$

SVAR: MINSTA VÄRDE: 0
STÖRSTA VÄRDE: $\frac{1}{2}e^{-1}$

---

2 a)
$$\int_0^{\pi} x\cos x\,dx = \left[x\sin x\right]_0^{\pi} - \int_0^{\pi}\sin x\,dx =$$

$$= \left[x\sin x + \cos x\right]_0^{\pi} = \pi\sin\pi + \cos\pi - (0 + \cos 0) =$$

$$= -1 - 1 = -2$$

SVAR: -2

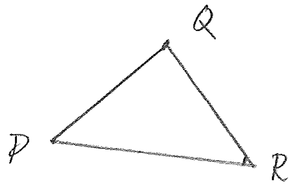
b)
$$f(x) = x\cos x^2$$

$$\int x\cos x^2\,dx = \left| \begin{matrix} x^2 = t \\ 2x\,dx = dt \end{matrix} \right| = \int \frac{1}{2}\cos t\,dt = \frac{1}{2}\sin t + C$$

$$= \frac{1}{2}\sin x^2 + C$$

SVAR: $\frac{1}{2}\sin x^2 + C$

3a)
$$P = (1,0,2) \qquad Q = (3,3,3) \qquad R = (4,-1,0)$$



$$\overrightarrow{PQ} = (3,3,3) - (1,0,2) = (2,3,1)$$

$$\overrightarrow{PR} = (4,-1,0) - (1,0,2) = (3,-1,-2)$$

$$\bar{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} e_x & e_y & e_z \\ 2 & 3 & 1 \\ 3 & -1 & -2 \end{vmatrix} = (-6+1,\ 3+4,\ -2-9)$$

$$= (-5, 7, -11) = -(5, -7, 11)$$

PLANET π: $\bar{n}\cdot(\bar{r} - \bar{r}_0) = 0 \Rightarrow (5,-7,11)(x-1, y, z-2) = 0$

$$5(x-1) - 7y + 11(z-2) = 0 \qquad 5x - 7y + 11z - 27 = 0$$: SVAR

3b) AREAN: $\frac{1}{2}|\vec{PQ} \times \vec{PR}| = \frac{1}{2}\sqrt{5^2+(-7)^2+11^2} =$

$= \frac{1}{2}\sqrt{25+49+121} = \frac{1}{2}\sqrt{195}$ SVAR! $\frac{1}{2}\sqrt{195}$ a.e

3c) $\begin{cases} x = 4 + 5t \\ y = -1 - 7t \\ z = 0 + 11t \end{cases}$ SVAR! $\begin{cases} x = 4+5t \\ y = -1-7t \\ z = 11t \end{cases}$

4. $y'' + 4y' + 13y = 26x - 5 \qquad y(0) = 10, \quad y'(0) = 40$

1) HOMOGEN LÖSNING: KAR. EKV: $k^2 + 4k + 13 = 0$

$k = -2 \pm \sqrt{4-13} \iff k = -2 \pm 3i$

$y_h = e^{-2x}(A\cos 3x + B\sin 3x)$

2) PART. LÖSNING: SÄTT $y = ax + b \Rightarrow y' = a, y'' = 0$

INSATT GER DET:

$4a + 13(ax+b) \equiv 26x - 5$

$\begin{cases} 13a = 26 \iff a = 2 \\ 4a + 13b = -5 \iff b = \frac{1}{13}(-5-4a) = \frac{1}{13}(-5-8) = -1 \end{cases}$

$\therefore y_p = 2x - 1$

$y(x) = 2x - 1 + e^{-2x}(A\cos 3x + B\sin 3x)$

$y'(x) = 2 - 2e^{-2x}(A\cos 3x + B\sin 3x) + e^{-2x}(-3A\sin 3x + 3B\cos 3x)$

$y(0) = -1 + A = 10 \iff A = 11$

$y'(0) = 2 - 2A + 3B = 40 \iff B = \frac{1}{3}(40 - 2 + 2\cdot 11) = 20$

$y(x) = 2x - 1 + e^{-2x}(11\cos 3x + 20\sin 3x)$

SVAR! $y(x) = 2x - 1 + e^{-2x}(11\cos 3x + 20\sin 3x)$

5.

$$P(x) = a + bx + cx^2 \qquad f(x) = \arctan 2x \quad \text{NÄRA } x=0$$

$$f(x) = f(0) + f'(0)\cdot(x-0) + \frac{f''(0)}{2}(x-0)^2 + O(x^3) = P(x) + O(x^3)$$

$$f(0) = \arctan 0 = 0$$

$$f'(x) = \frac{2}{1+(2x)^2} = \frac{2}{1+4x^2} \qquad f'(0) = 2$$

$$f''(x) = -2\cdot(1+4x^2)^{-2}\cdot 8x = \frac{-16x}{(1+4x^2)^2} \qquad f''(0) = 0$$

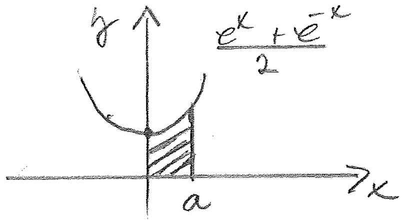
$$\therefore \quad P(x) = 0 + 2x + 0 \qquad \therefore \quad a=0,\ b=2,\ c=0$$

<u>SVAR</u>! $a = c = 0$, $b = 2$, $P(x) = 2x$

---

6. a)

$$y = \frac{e^x + e^{-x}}{2}$$



$$A = \int_0^a \frac{e^x + e^{-x}}{2}\,dx = \left[\tfrac{1}{2}(e^x - e^{-x})\right]_0^a = \tfrac{1}{2}\left(e^a - e^{-a} - (1-1)\right) =$$

$$= \tfrac{1}{2}(e^a - e^{-a}) \qquad A = 1 \Rightarrow \tfrac{1}{2}(e^a - e^{-a}) = 1$$

$$e^a - e^{-a} = 2 \quad |e^a = t| \Rightarrow t - \frac{1}{t} = 2$$

$$t^2 - 2t - 1 = 0 \Leftrightarrow t = 1 \pm \sqrt{2}, \quad e^a = 1 \pm \sqrt{2}$$

ENDAST $e^a = 1+\sqrt{2}$ ÄR DEF. $\Rightarrow a = \ln(1+\sqrt{2})$

<u>SVAR</u>: KOORDINATEN ÄR $(\ln(1+\sqrt{2}), 0)$

---

b)

$$V = \pi \int_0^{\ln(1+\sqrt{2})} \left(\frac{e^x + e^{-x}}{2}\right)^2 dx = \frac{\pi}{4}\int_0^{\ln(1+\sqrt{2})} \left(e^{2x} + 2 + e^{-2x}\right) dx$$

$$= \frac{\pi}{4}\left[\tfrac{1}{2}e^{2x} + 2x - \tfrac{1}{2}e^{-2x}\right]_0^{\ln(1+\sqrt{2})} =$$

$$= \frac{\pi}{4}\left(\tfrac{1}{2}e^{2\ln(1+\sqrt{2})} + 2\ln(1+\sqrt{2}) - \tfrac{1}{2}e^{-2\ln(1+\sqrt{2})} - \left(\tfrac{1}{2} - 0 - \tfrac{1}{2}\right)\right)$$

FORTS.

6b) FORTS.

$$\frac{\pi}{4}\left(\frac{1}{2}e^{\ln(1+\sqrt{2})^2} + 2\ln(1+\sqrt{2}) - \frac{1}{2}\cdot\frac{1}{e^{\ln(1+\sqrt{2})^2}}\right) =$$

$$= \frac{\pi}{4}\left(\frac{1}{2}(1+\sqrt{2})^2 + 2\ln(1+\sqrt{2}) - \frac{1}{2}\cdot\frac{1}{(1+\sqrt{2})^2}\right) =$$

$$= \frac{\pi}{4}\left(\frac{1}{2}(1+2\sqrt{2}+2) + 2\ln(1+\sqrt{2}) - \frac{1}{2}\frac{1}{1+2\sqrt{2}+2}\right) =$$

$$= \frac{\pi}{4}\left(2\ln(1+\sqrt{2}) + \frac{1}{2}\left((3+2\sqrt{2}) - \frac{1}{(3+2\sqrt{2})}\right)\right) =$$

$$= \frac{\pi}{4}\left(2\ln(1+\sqrt{2}) + \frac{1}{2}\left(3+2\sqrt{2} - \frac{(3-2\sqrt{2})}{\underbrace{9-4\cdot 2}_{=1}}\right)\right) =$$

$$= \frac{\pi}{4}\left(2\ln(1+\sqrt{2}) + \frac{1}{2}\cdot 4\sqrt{2}\right) = \frac{\pi}{2}\left(\ln(1+\sqrt{2}) + \sqrt{2}\right)$$

<u>SVAR</u>: $\frac{\pi}{2}\left(\ln(1+\sqrt{2})+\sqrt{2}\right)$ v.e

---

7.

$$\lim_{x\to 0}\frac{x\cos x - \sqrt{1+x^2}\sin x}{x^2\sin x} = \lim_{x\to 0}\frac{x\cos x - (1+x^2)^{\frac{1}{2}}\sin x}{x^2\sin x}$$

$$= \lim_{x\to 0}\frac{x\left(1-\frac{x^2}{2}+\frac{x^4}{4!}+O(x^6)\right) - \left(1+\frac{1}{2}x^2+\frac{1}{2}\left(\frac{1}{2}-1\right)\frac{x^4}{4\cdot 2}+O(x^6)\right)\left(x-\frac{x^3}{3!}+O(x^5)\right)}{x^2\left(x-\frac{x^3}{3!}+O(x^5)\right)}$$

$$= \lim_{x\to 0}\frac{x-\frac{x^3}{2}-\left(x-\frac{x^3}{6}+\frac{x^3}{2}\right)+O(x^4)}{x^3+O(x^4)} =$$

$$= \lim_{x\to 0}\frac{x^3\left(-\frac{1}{2}+\frac{1}{6}-\frac{1}{2}+O(x)\right)}{x^3(1+O(x))} = \lim_{x\to 0}\frac{\frac{1}{6}-1+O(x)}{1+O(x)}$$

$$= -\frac{5}{6}$$ (ALT. ANVÄND L'HOSPITALS REGEL)

<u>SVAR</u>: $-\frac{5}{6}$

8.

$$h(x) = \frac{x^2+x+1}{x+1} = \frac{x(x+1)+1}{x+1} = x + \frac{1}{x+1} \quad , \quad x \neq -1$$

ASYMPTOTER!

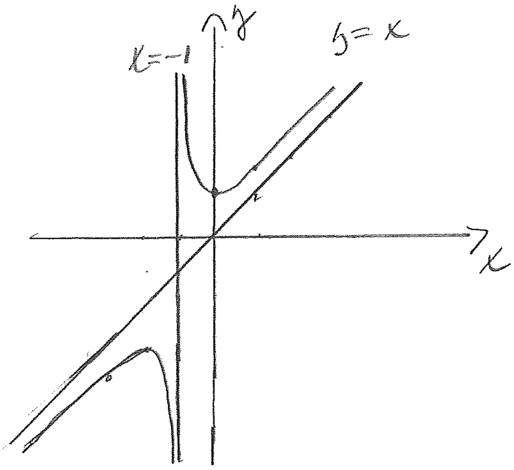
SNED AS : $y = x$

x- AS : $x = -1$

$$h'(x) = \frac{(2x+1)(x+1) - (x^2+x+1)}{(x+1)^2} = \frac{2x^2+3x+1-x^2-x-1}{(x+1)^2} =$$

$$= \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2} \qquad h'(x) = 0 \iff x=0,\ x=-2$$

x		-2		-1		0	
$h'(x)$	$+$	0	$-$	$\nexists$	$-$	0	$+$
h	$\nearrow$		$\searrow$	$\nexists$	$\searrow$		$\nearrow$
		LOK MAX				LOK MIN	



LOK. MAX PUNKT I $(-2, -3)$

LOK MIN. PUNKT I $(0, 1)$

$h(x)$ ÄR VÄXANDE DÅ $x < -2$ OCH $x > 0$

" " AVTAGANDE DÅ $-2 < x < -1$ OCH $-1 < x < 0$.

9.

$$y = x \sin x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad P(a, b)$$

$$y' = \sin x + x\cos x \qquad k_T(a) = \sin a + a\cos a$$

$$k_N = \frac{-1}{k_T} \qquad k_N(a) = \frac{-1}{\sin a + a\cos a}$$

NORMALEN I (a,b) HAR EKV:

$$y_N - b = \frac{-1}{\sin a + a\cos a}(x-a) \iff y_N = b + \frac{a-x}{\sin a + a\cos a}$$

$$b = a\sin a \implies y_N = a\sin a + \frac{a-x}{\sin a + a\cos a}$$

FORTS.

NORMALENS Y-VÄRDE DÅ $x=0$:

$$y_N = a\sin a + \frac{a}{\sin a + a\cos a} = N(a)$$

$$\lim_{a\to 0} N(a) = \lim_{a\to 0}\left(a\sin a + \frac{a}{\sin a + a\cos a}\right) =$$

$$= \lim_{a\to 0}\left(a\sin a + \frac{a}{a\left(\frac{\sin a}{a} + \cos a\right)}\right) =$$

$$= \lim_{a\to 0}\left(a\sin a + \frac{1}{\frac{\sin a}{a} + \cos a}\right) = 0 + \frac{1}{1+1} = \frac{1}{2}$$

<u>SVAR</u>: $\lim_{a\to 0} N(a) = \frac{1}{2}$

10.

$$\frac{1}{\sqrt{x}} + \ln x > 1000 \quad \text{DÅ} \quad x < 1 \ ?$$

$f(x) = \frac{1}{\sqrt{x}} + \ln x$, $x > 0$ ÄR $f(x) > 1000$ DÅ $x < 1$?

$$f'(x) = -\frac{1}{2x\sqrt{x}} + \frac{1}{x} = \frac{2\sqrt{x}-1}{2x\sqrt{x}}, \quad x > 0$$

$$f'(x) = 0 \Rightarrow 2\sqrt{x} - 1 = 0$$
$$\sqrt{x} = \frac{1}{2} \iff x = \frac{1}{4}$$

x	0		$\frac{1}{4}$		1
$f'(x)$	$\nexists$	$-$	0	$+$	
$f(x)$	$\nexists$	$\searrow$	LOK MIN	$\nearrow$	

$f(\frac{1}{4}) = \frac{1}{\frac{1}{2}} + \ln\frac{1}{4} = 2 - \ln 4 =$
$= 2 - \ln 2^2 = 2 - 2\ln 2 =$
$= 2(1 - \underbrace{\ln 2}_{>0}) > 0.$

$f(\frac{1}{4}) > 0$ ÄR MINSTA VÄRDET FÖR $f(x)$.

UNDERSÖK INTERVALLGRÄNSERNA:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{1}{\sqrt{x}} + \ln x = 1 + 0 = 1.$$

FORTS

10. FORTS.

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{1}{\sqrt{x}} + \ln x = \lim_{x \to 0^+} \frac{1 + \sqrt{x}\ln x}{\sqrt{x}} =$$

$$= \frac{1+0}{0} \to +\infty \qquad \lim_{x \to 0^+} x^a \ln x \to 0$$ STANDARD-GRÄNSVÄRDE $(a > 0)$

∴ DET FINNS TAL NÄRA 0 SOM GER $f(x) > 1000$.

SVAR: JA, FÖR TILLRÄCKLIGT SMÅ $x > 0$ ÄR $\frac{1}{\sqrt{x}} + \ln x > 1000$.