Problem 73# 1.2 Solution

Suppose the sets $A(a_i, d_i)$ are pairwise disjoint , and their union comprises all sufficiently large integers. We associate to the set $A(a, d)$the power series $\sum z^j$, where j ranges over $A(a, d)$, that is, the function

$$\sum_{k=0}^{\infty} z^{a+kd} = z^a/(1-z^d).$$

Then our hypotheses imply that

$\sum_{i=1}^{m} z^{a_i}/(1-z^{d_i})$, (where m denotes the number of arithmetic progressions)

is equal to

$\frac{1}{1-z}$ plus a polynomial in z.

If we assume d_m is larger than all of the d_i with $i < m$, then we deduce from this identity a contradiction by letting z tend to a point of the unit circle which is a primitive root of unity of order d_m.
Indeed, the term $z^{a_m}/(1-z^{d_m})$ is then unbounded, while all the other terms tend to finite limits.

HSS