Problem73# 3.1 Solution

Here the generalized arithmetic-geometric inequality (see also the solution to 73#1.1) becomes useful:

$$\prod_{j=1}^{n}(x_j)^{p_j} \le \sum_{j=1}^{n} p_j x_j$$

(where $x_j > 0$ and $\sum p_j = 1$).
Careful choices of the parameters p_j will do the job $(x, y, z > 0$):
$3 = xy + yz + xz = 9 \left(\frac{1}{9}xy + \frac{3}{9}(xz/3) + \frac{5}{9}(yz/5)\right) \ge$

$\ge 9(xy)^{1/9} \cdot (xz/3)^{1/3} \cdot (yz/5)^{5/9} = \frac{9}{(3^3 5^5)^{1/9}} \cdot (x^2y^3z^4)^{2/9}.$
Obviously maximum is attained where equality occurs i.e. when
$(*) \quad xy = xz/3 = yz/5 \quad$ and $\quad (**) \quad xy + yz + zx = 3$.
From $(*) : z = 5x,\ y = 5x/3$ and $(**) : (5/3 + 25/3 + 5)x^2 = 3$ which yields $x^2 = 1/5$
and $x = \frac{1}{\sqrt{5}},\ y = \frac{\sqrt{5}}{3},\ z = \sqrt{5}.$

BE/GJ