Problem 73# 3.3 Solution

Problem 3: Prove the equation $x^5 + ax^4 + 10x^3 + cx^2 + dx + e = 0$ with $|e| > 1$ can't have 5 real roots of like sign.

Proof. Suppose it has 5 positive roots r_j, j from 1 to 5. The sum of the 10 products $r_i r_j$ with $1 \leq i < j \leq 5$ equals 10, so the geometric mean of these cannot exceed 1, which gives that the product of all r_i cannot exceed 1. But this product equals $|e| > 1$, contradiction.

The case of 5 negative roots is disposed of similarly.

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