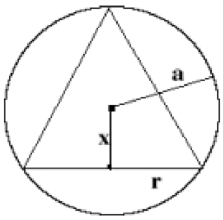
Problem73# 3.4 Solution



With notation according to figure we get:
Cone height, $h = a + x$.
Cone base radius $r = \sqrt{a^2 - x^2}$.
Lateral area of cone, Ł $= \pi r \sqrt{r^2 + h^2} =$
$\pi \sqrt{a^2 - x^2} \cdot \sqrt{a^2 - x^2 + (a + x)^2} =$
$\pi \sqrt{2a} \sqrt{(a - x)(a + x)^2}$.

The straight AG-inequality (case $n = 3: \ (a_1 + a_2 + a_3)/3 \geq \sqrt[3]{a_1 a_2 a_3}$) gives:

$$\frac{2a}{3} = \frac{1}{3}(a - x + (a + x)/2 + (a + x)/2) \geq \sqrt[3]{\tfrac{1}{4}(a - x)(a + x)^2} = kL^{2/3}.$$

(Note that x may be negative, but $|x| \leq a$.)
The maximum value of L corresponds to the case of equality in the AG inequality.
Hence $a - x = (a + x)/2$ i.e. $x = a/3$.
The inscribed cone with maximal lateral area has height $\underline{h = 4a/3.}$

BE/GJ