Set # 3

Problem 73# 3.6 Solution

Problem 6. For each pair of distinct integers i, j from $\{1, 2, ..., n\}$ we have $(a_i - a_j)(b_i - b_j) \geq 0$. Expanding, we get

$$a_i b_j + a_j b_i \leq a_i b_i + a_j b_j. \qquad (*)$$

Let A denote the sum of the a_i, and B the sum of the b_i. Summing (*) over i,

$$A b_j + a_j B \leq \sum [a_i b_i] + n\, a_j b_j.$$

Now sum over j:

$$AB + AB \leq n \sum [a_i b_i] + n \sum [a_j b_j], \quad \text{or}$$

$$AB \leq n \sum [a_i b_i]$$

which is the desired inequality, after multiplying each member by $1/n^2$.

Remark. This is a standard inequality, due to Chebyshev.

HSS