1) $\left(\begin{array}{ccc|c} 1 & 3 & 2 & 3 \\ 2 & 8 & 3 & -2 \\ 3 & 10 & 6 & 7 \end{array}\right) \xrightarrow{-3} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 3 \\ 2 & 8 & 3 & -2 \\ 0 & 1 & 0 & -2 \end{array}\right) \xrightarrow{-3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 2 & 8 & 3 & -2 \\ 0 & 1 & 0 & -2 \end{array}\right) \xrightarrow{-8}$

$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 2 & 0 & 3 & 14 \\ 0 & 1 & 0 & -2 \end{array}\right) \xrightarrow{-2} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -2 \end{array}\right) \xrightarrow{2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -2 \end{array}\right)$

$\Rightarrow x = 1, \ y = -2, \ z = 4$

(Alternativt med hjälp av Cramer's regel: $\begin{vmatrix} 1 & 3 & 2 \\ 2 & 8 & 3 \\ 3 & 10 & 6 \end{vmatrix} = 1$, $\begin{vmatrix} 3 & 3 & 2 \\ -2 & 8 & 3 \\ 7 & 10 & 6 \end{vmatrix} = 1$, $\begin{vmatrix} 1 & 3 & 2 \\ 2 & -2 & 3 \\ 3 & 7 & 6 \end{vmatrix} = -2$ ...)

2) $f(\vec{x}) = (\vec{x}-\vec{a})\cdot(\vec{x}-\vec{b}) \Rightarrow \vec{\nabla} f = (\vec{x}-\vec{b}) + (\vec{x}-\vec{a})$

$\vec{\nabla} f \overset{!}{=} \vec{0} \Rightarrow \vec{x}_0 = \frac{1}{2}(\vec{a}+\vec{b})$ (möjligt extremvärde)

$H(\vec{x}) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = H(\vec{x}_0)$ positivt definit

$\Rightarrow$ Minimum! (och $f(\vec{x}_0) = \left(\frac{\vec{b}}{2}-\frac{\vec{a}}{2}\right)\left(\frac{\vec{a}}{2}-\frac{\vec{b}}{2}\right) = -\frac{1}{4}(\vec{a}-\vec{b})^2$)

$(\vec{a}-\vec{b})^2 = |\vec{b}-\vec{a}|^2 = \left|\begin{pmatrix} 2 \\ 2 \end{pmatrix}\right|^2 = 4+4 \Rightarrow f(\vec{x}_0) = -2$

4) $\vec{\nabla} f(\vec{0}) = \begin{pmatrix} \cos x + 2xy \\ x^2 \\ 2e^{2z} \end{pmatrix}\Bigg|_{\vec{x}=\vec{0}} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \vec{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$\vec{m} = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow f'_{\vec{m}}(\vec{0}) = \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \frac{4}{\sqrt{5}} = \vec{m}\cdot\vec{\nabla} f(\vec{0})$