(3)

6) $\vec{\nabla} F = \begin{pmatrix} \frac{2x}{a^2} \\ \frac{2y}{b^2} \\ \frac{2z}{c^2} \end{pmatrix}$, $P = \left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}, 0\right)$ $\Longrightarrow$ $\vec{\nabla} F(P) = \begin{pmatrix} \frac{\sqrt{2}}{a} \\ \frac{\sqrt{2}}{b} \\ 0 \end{pmatrix}$

$F(\vec{x}) := \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 \overset{!}{=} 0$

$\Longrightarrow$ tangentplanet är given som $\frac{\sqrt{2}}{a} x + \frac{\sqrt{2}}{b} y + 0 \cdot z \underset{(x,y,z) = \left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}, 0\right)}{=} 1 + 1 + 0 = 2$

(dvs $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$)

7) $|J_f| = \begin{vmatrix} x_r & x_\varphi \\ y_r & y_\varphi \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r > 0$

$J_{f^{-1}} = (J_f)^{-1} = \frac{1}{r} \begin{pmatrix} r\cos\varphi & r\sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \underset{(x,y) = (2,0)}{=} \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

$\Longrightarrow r = 2, \varphi = 0$

8) $\begin{vmatrix} -5-\lambda & 4 & 2 \\ 4 & -5-\lambda & 2 \\ 2 & 2 & -8-\lambda \end{vmatrix} = -(5+\lambda)\left[(5+\lambda)(8+\lambda) - 4\right] - 4\left[4(-8-\lambda) - 4\right] + 2\left[8 + 2(5+\lambda)\right]$

$= -(5+\lambda)\left[\lambda^2 + 13\lambda + 36\right] + 4\left[4\lambda + 36\right] + 4\left[\lambda + 9\right]$

$= -\lambda^3 + \lambda^2(-5-13) + \lambda(-13 \cdot 5 - 36 + 16 + 4) + (-5 \cdot 36 + 4 \cdot 36 + 36)$

$= -\lambda^3 - 18\lambda^2 - 81\lambda + 0 = -\lambda(\lambda^2 + 18\lambda + 81) = -\lambda(\lambda+9)^2$

(normerad) egenvektor till $\lambda = 0$:

$\begin{pmatrix} -5 & 4 & 2 \\ 4 & -5 & 2 \\ 2 & 2 & -8 \end{pmatrix} \vec{x}_0 = \vec{0} \Longrightarrow \vec{x}_0 = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$;

egenvektor till $\lambda = -9$:

$\begin{pmatrix} 4 & 4 & 2 \\ 4 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{0} \Longleftrightarrow 2x + 2y + z = 0$, t.ex $\vec{x}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$