A symmetrisk $\Rightarrow$ det existerar 3 vinkelräta egenvektorer (4)

$\Rightarrow$ man kan beräkna $\vec{x}_2$ som

$$\vec{x}_2 := \vec{x}_0 \times \vec{x}_1 = \frac{1}{3\cdot\sqrt{2}}\begin{pmatrix}2\\2\\1\end{pmatrix}\times\begin{pmatrix}1\\-1\\0\end{pmatrix} = \frac{1}{\sqrt{18}}\begin{pmatrix}1\\1\\-4\end{pmatrix}$$

(OBS: $\frac{1}{\sqrt{18}}(2\cdot 1 + 2\cdot 1 - 4) = 0$, dvs $A\vec{x}_2 = -9\vec{x}_2$!)

$\Rightarrow A = R\cdot\begin{pmatrix}0&0&0\\0&9&0\\0&0&9\end{pmatrix}\cdot R^T$, då $R = (\vec{x}_0\,\vec{x}_1\,\vec{x}_2)$ (t.ex.)

9) $x^2 - 4x + y^2 - 6y + 12 = 0 \Leftrightarrow (x-2)^2 + (y-3)^2 = 1$

Parametrisering: $x(\varphi) = 2 + \cos\varphi$, $y(\varphi) = 3 + \sin\varphi$

$\Rightarrow \tilde{f}(\varphi) := f(\vec{x}(\varphi)) = 4\cos^2\varphi + \sin^2\varphi = 1 + 3\cos^2\varphi$;

$\tilde{f}'(\varphi) \overset{!}{=} 0$ $(-6\sin\varphi\cos\varphi = 0) \Longrightarrow \varphi = \varphi_j = j\cdot\pi/2$ $(j = 0,1,2,3)$

$\underbrace{\tilde{f}(\varphi_0) = \tilde{f}(\varphi_2) = 4}_{MAX}$, $\underbrace{\tilde{f}(\varphi_1) = \tilde{f}(\varphi_3) = 1 + 3\cdot 0 = 1}_{MIN} \Longrightarrow$

(Alternativt: $F(x,y,\lambda) := f(x,y) - \lambda(x^2 - 4x + y^2 - 6y + 12)$)
Lagrange's metod $\quad \nabla F = \vec{0}$ ...

10) $S^2 = -E \Rightarrow S^3 = -S$, $S^4 = E$, ...

$R(\varphi)\cdot R(\theta) = e^{\varphi S}\cdot e^{\theta S} = e^{(\varphi+\theta)S} = R(\varphi+\theta)$

(alternativt: additionsteoremen + matrismultiplikation)

$R(\varphi)\big(R(\theta)\cdot R(-\varphi)\big) = R(\varphi)\,R(\theta-\varphi) = R(\varphi+\theta-\varphi) = R(\theta)$

$\cos\varphi = 1 - \varphi^2/2! + \varphi^4/4! - \ldots$, $\sin\varphi = \varphi - \varphi^3/3! + \ldots$