5B1121 Matematik baskurs. Tentamen 13/1 2006. Lösn.

(1) $\frac{1}{x} + \frac{1}{x-2} - \frac{2}{x(x-2)} = 6$ | MGN: $x(x-2)$ | $x \neq 0$, $x \neq 2$

$\Rightarrow (x-2) + x - 2 = 6x(x-2)$

$\Rightarrow 2x - 4 = 6x^2 - 12x$

$\Rightarrow 6x^2 - 12x - 2x + 4 = 0$

$\Rightarrow 6x^2 - 14x + 4 = 0$

$\Rightarrow x^2 - \frac{14}{6}x + \frac{4}{6} = 0$

$\Rightarrow x - \frac{7}{3}x + \frac{2}{3} = 0$

$\Rightarrow x = \frac{7}{6} \pm \sqrt{\frac{49}{36} - \frac{2 \cdot 12}{3 \cdot 12}}$

$\Rightarrow x = \frac{7}{6} \pm \sqrt{\frac{25}{36}}$

$\Rightarrow x = \frac{7}{6} \pm \frac{5}{6}$,

$x = \frac{12}{6} = 2$ EJ SAT, TY $x \neq 2$

$x = \frac{2}{6} = \frac{1}{3}$ SVAR: $x = 1/3$

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(2) $\ln x - \ln \frac{2}{x} = \ln(x+4)$, $x > 0$

$\Rightarrow \ln \frac{x}{\frac{2}{x}} = \ln(x+4)$

$\Rightarrow \ln \frac{x^2}{2} = \ln(x+4)$

$\Rightarrow \frac{x^2}{2} = x + 4$

$\Rightarrow x^2 = 2x + 8$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = 1 \pm \sqrt{1+8}$

$\Rightarrow x = 1 \pm 3$

$\Rightarrow x = 4$, $x = -2$ EJ DEF TY $x > 0$

SVAR: $x = 4$

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(3) $(x - 2x^2)^6 = \binom{6}{0}x^6 + \binom{6}{1}x^5 \cdot (2x^2)^1 + \binom{6}{2}x^4 \cdot (-2x^2)^2 + \binom{6}{3}x^3(-2x^2)^3$

$+ \binom{6}{4}x^2(-2x^2)^4 + \ldots, \binom{6}{4} = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2 \cdot 1} = 3 \cdot 5 = 15$

$15x^2 \cdot (-2)^4 \cdot x^8 = 15 \cdot 16 x^{10}$ KOEFF ÄR $15 \cdot 16 = 240$,

SVAR: 240

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(4) $\binom{n}{2} \leq \binom{n+1}{3}$ $n = 2, 3, \ldots$

VISA $\binom{n}{2} - \binom{n+1}{3} \leq 0$

$\frac{n!}{(n-2)!\,2} - \frac{(n+1)!}{(n+1-3)!\cdot 3!} = \frac{n \cdot (n-1)}{2} - \frac{(n+1) \cdot n(n-1)}{6} =$

($(n-2)!$)

$= \frac{n(n-1)}{2}\left(1 - \frac{n+1}{3}\right) = \frac{n(n-1)}{2}\left(\frac{3-(n+1)}{3}\right) = \frac{n(n+1)(2-n)}{2 \cdot 3} \leq 0$

DÅ $n = 2, 3, \ldots$, TY $n > 0$, $n+1 > 0$ OCH $(2-n) \leq 0$

DÅ $n = 2, 3, \ldots$ VSV.

(5) $\cos\left(\frac{\pi}{2}-2x\right) = \sin 2x$ och $\sin 2x = 1 \iff 2x = \frac{\pi}{2} + n2\pi$

$\iff x = \frac{\pi}{4} + n\pi$, n heltal.

Eftersom $\tan\left(\frac{\pi}{4} + n\pi\right) > 0$ för alla heltalen så finns inga tal x som uppfyller villkoren.

SVAR: Inga.

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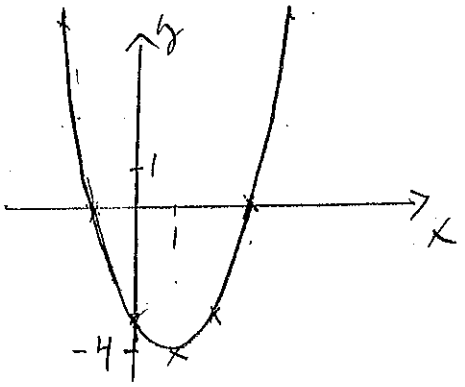
6. a) $x^2 - 2x - 3 = (x-1)^2 - 1 - 3 = (x-1)^2 - 4$.

$f(x) = (x-1)^2 - 4$ ÄR MINST DÅ $x = 1$ OCH MINSTA VÄRDET ÄR $f(1) = -4$.

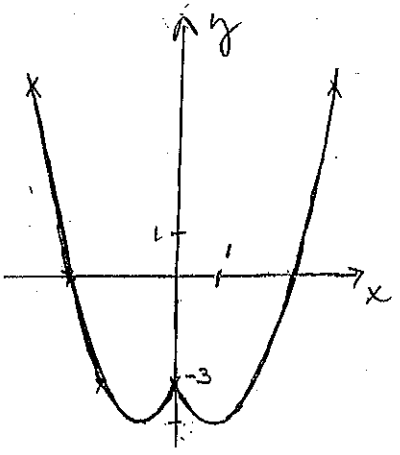
b)

x	$y = x^2 - 2x - 3$
1	-4
2	$1-4=-3$
0	-3
3	0
-1	0
4	5
-2	5



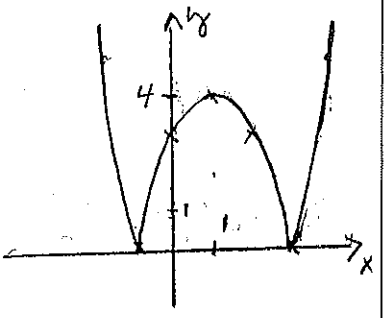
6c) $y = |x|^2 - 2|x| - 3$

x	y
1	-4
-1	-4
0	-3
± 2	-3
± 3	0
± 4	5



6d) $y = |x^2 - 2x - 3|$

x	y
1	$\lvert -4\rvert = 4$
2	$\lvert -3\rvert = 3$
0	$\lvert -3\rvert = 3$
-1	0
3	0
4	5
-2	5



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7. $\cosh x = \frac{e^x + e^{-x}}{2}$ $\quad \sinh x = \frac{e^x - e^{-x}}{2}$

a) $(\cosh x)^2 - (\sinh x)^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} =$

$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} = \frac{4}{4} = 1$

b) $2\sinh x \cosh x = 2 \cdot \left(\frac{e^x - e^{-x}}{2}\right) \cdot \left(\frac{e^x + e^{-x}}{2}\right) =$

$= \frac{e^{2x} + 1 - 1 - e^{-2x}}{2} = \frac{e^{2x} - e^{-2x}}{2} = (\sinh 2x)$

7c $(\cosh x)^2 + (\sinh x)^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2 =$

$= \frac{e^{2x} + 2 + e^{-2x}}{4} + \frac{e^{2x} - 2 + e^{-2x}}{4} = \frac{2e^{2x} + 2e^{-2x}}{4} =$

$= \frac{e^{2x} + e^{-2x}}{2} = (\cosh 2x)$. SVAR: a) 1 b) $\sinh 2x$ c) $\cosh 2x$.

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8. ARITM. TALFÖLJD $a, a+d, a+2d,$

GEOM. TALFÖLJD $a, ak, ak^2, \ldots$

VILLKOR: I) $a + a+d + a+2d = 39$

OCH II) $a, a+d-1, a+2d$ ÄR GEOM. TALF.

I) $3a + 3d = 39 \iff a+d = 13$

II) $\frac{a+d-1}{a} = \frac{a+2d}{a+d-1}$

SÄTT IN $a+d = 13$ I II:

$\frac{13-1}{a} = \frac{13+d}{13-1} \iff \frac{12}{a} = \frac{13+d}{12}$

$12 \cdot 12 = (13+d)a$

$144 = (13+d)a \quad | a = 13-d$ ur I |

$144 = (13+d)(13-d)$

$144 = 169 - d^2 \iff d^2 = 169 - 144$

$d^2 = 25 \quad d = \pm 5 \quad d=5 \Rightarrow a=8$

$d=-5 \Rightarrow a=18$

! TALEN ÄR 8, 13, 18 ELLER 18, 13, 8

SVAR! 8, 13, 18 ELLER 18, 13, 8