# Repetition

Övn: Förenkla

$$(A^T B)^{-1} (C^{-1} A)^T (B^T C)^T$$

$$= B^{-1} \underbrace{(A^T)^{-1} A^T}_{E} (C^{-1})^T C^T (B^T)^T$$

$$= B^{-1} \underbrace{(C^T)^{-1} C^T}_{E} B = B^{-1} B = E$$

Berikut

$$\det\left(A^T (BA^{-1})^T (C^T B)^{-1}\right)$$

$$\boxed{\det(XY) = \det(YX) = \det X \cdot \det Y}$$

$$= \det A^T \det (BA^{-1})^T \det (C^T B)^{-1}$$

$$= \det A \det B \det A^{-1} \det (C^T B)^{-1}$$

$$= \cancel{\det A} \cancel{\det B} \cancel{\frac{1}{\det A}} \frac{1}{\det C} \cancel{\frac{1}{\det B}}$$

$$\boxed{\det A^T = \det A}$$

Berikan

$$\begin{vmatrix} 3 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ -1 & 0 & 1 & -1 \end{vmatrix} = (-1)\begin{vmatrix} 3 & 0 & -1 \\ 1 & -1 & 1 \\ -1 & 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 3 & 0 & -1 \\ 1 & -1 & 1 \\ 0 & 2 & 0 \end{vmatrix}$$

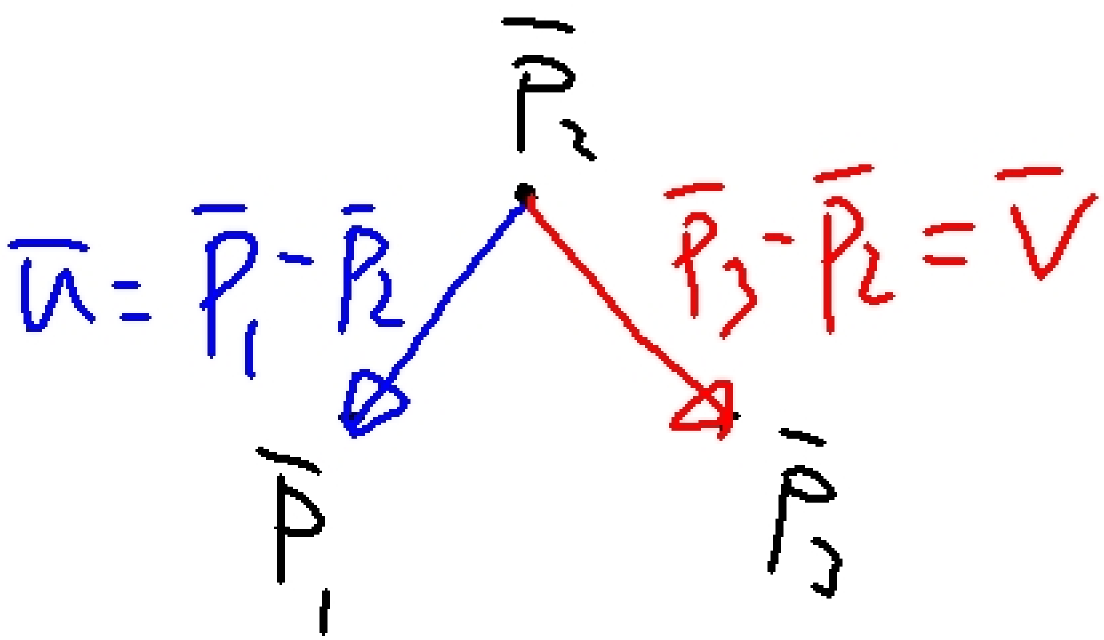
$$= (-1)(3+1) - (-2-6) = 4$$

$$\left|\begin{matrix} 3 & 0 & -1 \\ 1 & -1 & 1 \\ -1 & 0 & -1 \end{matrix}\right| \begin{matrix} 3 & 0 \\ 1 & -1 \\ -1 & 0 \end{matrix}$$

$$\bar{u} = (3,0,1) \quad , \quad \bar{v} = (2,-1,1)$$

$$\bar{u} \cdot \bar{v} = 3 \cdot 2 + 0 \cdot (-1) + 1 \cdot 1 = 7$$

$$\bar{u} \times \bar{v} = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ 3 & 0 & 1 \\ 2 & -1 & 1 \end{vmatrix} \end{array} = (1, -1, -3)$$




$$\bar{n} = \bar{u} \times \bar{v}$$

Låt $\bar{u}=(7,3)$ bestäm en vektor vinkelrät mot $\bar{u}$.

SVAR: t ex $\bar{v}=(3,-7)$ för $\bar{v}\cdot\bar{u}=0$.

Lös ekvationen: $z^3=8i$

$z=re^{i\theta} \Rightarrow z^3=r^3e^{i3\theta}$

$8i=8e^{i\frac{\pi}{2}}$ $\quad r^3=8 \Rightarrow r=2$

$e^{i\theta}=\cos\theta+i\sin\theta$ $\quad 3\theta=\frac{\pi}{2}+2\pi n \Leftrightarrow \theta=\frac{\pi}{6}+\frac{2\pi n}{3}$

$\theta_0=\frac{\pi}{6}$, $\theta_1=\frac{\pi}{6}+\frac{2\pi}{3}=\frac{5\pi}{6}$, $\theta_2=\frac{\pi}{6}+\frac{4\pi}{3}=\frac{9\pi}{6}=\frac{3\pi}{2}$

$z_1=2e^{i\frac{\pi}{6}}=$

$=2\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$

$=\sqrt{3}+i$

$z_2=-\sqrt{3}+i$

$z_3=-2i$

$$\frac{2}{1-i}\cdot\frac{(1+i)}{(1+i)}=\frac{2+2i}{2}=1+i$$

Skriv linjen $(1,2)+t(-1,1)$ på normalform. $(ax+by=c)$

$(a,b)$ ska vara vinkelrät mot $(-1,1)$ dvs exvis $(1,1)$

$$x+y=c$$

$$1+2=3 \Rightarrow c=3.$$

SVAR: $x+y=3$

Visa med hjälp av induktion att

$2^n > n$, för $n \geq 1$.

Ind bas: $2^1 = 2 > 1$ okej

Ind ant: $2^p > p$

Ind steg: $2^{p+1} = 2 \cdot 2^p > 2p = p + p \geq p+1$

okej

Enligt induktionsprincipen gäller därför påståendet för alla $n \geq 1$.

$$D\sin^2(x^3) = 2\sin(x^3)\cos(x^3)3x^2$$

$f(g(h(x)))$ dir $\quad f(x) = x^2$

$g(x) = \sin x$

$h(x) = x^3$

$$D\sin^2(x^3) = 2\sin x^3 \, D(\sin(x^3)) =$$

$$= 2\sin(x^3)\cos(x^3)\cdot 3x^2$$

$$D(\sin(x^3)\cdot\sin(x^3)) = (D\sin(x^3))\sin x^3 + \sin x^3 D\sin x^3$$

$$D\left(\frac{\sin x}{\cos x}\right) = \frac{(D\sin x)\cos x - \sin x \, D\cos x}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

Bestäm konstanten $a$ så att

$$f(x) = \begin{cases} e^{x^2} & x < 0 \\ \sin(x+a) & x \geq 0 \end{cases}$$

$\lim_{x \to 0} e^{x^2} = 1$

Tex: $a = \frac{\pi}{2}$.

blir kontinuerlig.

$$\lim_{x \to 0} \sin(x+a) = \sin a = f(a)$$

Låt $f(x) = \begin{cases} e^{\frac{1}{x}} & x<0 \\ x^2 & x>0 \end{cases}$

Bestäm $f(0)$ så att $f$ utvidgas till en kontinuerlig funktion.

$$\lim_{x \to 0^-} e^{\frac{1}{x}} = \lim_{s \to -\infty} e^{+s} = 0$$

$$\lim_{x \to 0} x^2 = 0 \qquad \underline{f(0) = 0}$$

Bestäm Maclaurinpolynomet av grad 5 till

$$e^{\sin(x^2)}$$

$$e^t = 1 + t + \frac{t^2}{2!} + \dots$$

$$\sin t = t + O(t^3)$$

$$\sin(x^2) = x^2 + O(x^6)$$

$$e^{\sin(x^2)} = 1 + \left(x^2 + O(x^6)\right) + \frac{\left(x^2 + O(x^6)\right)^2}{2!} + O(x^6)$$

$$= 1 + x^2 + \frac{x^4}{2!} + O(x^6)$$

SVAR:

$$1 + x^2 + \frac{x^4}{2!}$$

$$x^2+y^2=6x$$

$$x^2-6x+y^2=0$$

$$(x-3)^2-9+y^2=0$$

$$(x-3)^2+y^2=9$$

Beräkna gränsvärdet

$$\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^3} =$$

$$= \lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2!}\right)\left(x - \frac{x^3}{3!}\right) + O(x^4) - x - x^2}{x^3} =$$

$$= \lim_{x \to 0} \frac{\frac{x^3}{2!} - \frac{x^3}{3!} + O(x^4)}{x^3} = \frac{1}{2!} - \frac{1}{3!} = \frac{1}{3}.$$

Alt L'Hôpital: $= \lim_{x \to 0} \frac{e^x \sin x + e^x \cos x - 1 - 2x}{3x^2} =$

$$= \lim_{x \to 0} \frac{\cancel{e^x \sin x} + e^x \cos x + e^x \cos x - \cancel{e^x \sin x} - 2}{6x} =$$

$$= \lim_{x\to 0} \frac{2e^x\cos x - 2}{6x} = \lim_{x\to 0} \frac{2e^x\cos x - 2e^x\sin x}{6} =$$

$$= \frac{2}{6} = \frac{1}{3}$$

Beräkna: $\int_1^3 x\ln(x^2)\,dx = \left[\begin{array}{l} t = x^2 \\ dt = 2x\,dx \end{array}\right] =$

$$= \frac{1}{2}\int_1^9 1\cdot\ln t\,dt = \frac{1}{2}\Big[t\ln t\Big]_1^9 - \frac{1}{2}\int_1^9 t\,\frac{1}{t}\,dt =$$

$$= \frac{1}{2}9\ln 9 - \underbrace{\frac{1}{2}1\ln 1}_{=0} - \frac{1}{2}\underbrace{(9-1)}_{=8} = \frac{1}{2}9\ln 9 - 4$$

Bestäm allmänna lösningen till

$$y'' + 3y' - 4y = \cos x$$

$$y = y_h + y_p$$

$$y_h'' + 3y_h' - 4y_h = 0$$

$$r^2 + 3r - 4 = 0 \qquad r = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}} = \begin{cases} 1 \\ -4 \end{cases}$$

$$y_h = Ae^{x} + Be^{-4x}$$

Ansatz

$$y_p = \underline{a}\cos x + \underline{b}\sin x$$

$$y_p' = -a\sin x + b\cos x$$

$$y_p'' = -a\cos x - b\sin x$$

$$\cos x = y_p'' + 3y_p' - 4y_p = -a\cos x - b\sin x + 3(-a\sin x + b\cos x) - 4(a\cos x + b\sin x)$$

$$= (\underbrace{-a + 3b - 4a}_{=1})\cos x + (\underbrace{-b - 3a - 4b}_{=0})\sin x$$

$$\begin{cases} -5a + 3b = 1 \\ -3a - 5b = 0 \Rightarrow b = -\frac{3a}{5} \end{cases} \qquad -\frac{34}{5}a = 1$$

$$y = Ae^{x} + Be^{-4x} - \frac{5}{34}\cos x + \frac{3}{34}\sin x$$