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Lösningsförslag till kontrollskrivning 3A
i 5B1147 Envariabelanalys för E, ht 2006.

1.

$$\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}\int_0^1 \frac{dx}{\sqrt{1-(x/2)^2}} = [\arcsin x/2]_0^1$$
$$= \arcsin(1/2) - 0 = \frac{\pi}{6}.$$

2.

$$\int_0^{\pi/2} (\sin x)^3\, dx = \int_0^{\pi/2} (\sin x)^2 \cdot \sin x\, dx = \int_0^{\pi/2} (1-(\cos x)^2)\cdot \sin x\, dx$$
$$= \{u = \cos x,\ du = -\sin x\, dx\} = \int_1^0 (1-u^2)(-du)$$
$$= \int_0^1 (1-u^2)\, du = \left[u - \frac{u^3}{3}\right]_0^1 = \frac{2}{3}.$$

3.
$$\int \frac{3x-9}{x^2-7x+10}\, dx = ?$$

$$x^2 - 7x + 10 = 0 \iff x = \frac{7}{2} \pm \sqrt{\frac{49-40}{4}} = \frac{7\pm 3}{2} = \begin{cases} 5 \\ 2 \end{cases}$$
$$\implies x^2 - 7x + 10 = (x-2)(x-5).$$

Så
$$\int \frac{3x-9}{x^2-7x+10}\, dx = \int \frac{3x-9}{(x-2)(x-5)}\, dx = \{\text{handpåläggning}\}$$
$$= \int \left(\frac{1}{x-2} + \frac{2}{x-5}\right) dx = \ln|x-2| + 2\ln|x-5| + C.$$