Dagens teman

- Integraler av harmoniska funktioner (Arb 5, §7.1)
- Faltning (§7.2)
- Fouriertransformen (§7.3)

Sinus cardinalis:

- $$\int_{-P/2}^{P/2} \mathrm{e}^{2\pi \mathrm{i} f t}\, \mathrm{d}f = \frac{\sin P\pi f}{\pi f} = P \operatorname{sinc} Pf,$$

- $$\int_{-P/2}^{P/2} \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}\omega = \frac{\sin P\omega/2}{\omega/2} = P \operatorname{sinc} \left(P \frac{\omega}{2\pi} \right)$$

δ-*pulsen som summa av alla harmoniska signaler*:

- $$\int_{-\infty}^{\infty} \mathrm{e}^{2\pi \mathrm{i} f t}\, \mathrm{d}f = \delta(t),$$

- $$\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}\omega = 2\pi\, \delta(t),$$

Fouriertransformen

Syntesekvationen:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{i\omega t} d\omega$$

Analysekvationen:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t} dt$$

Parsevals formel:

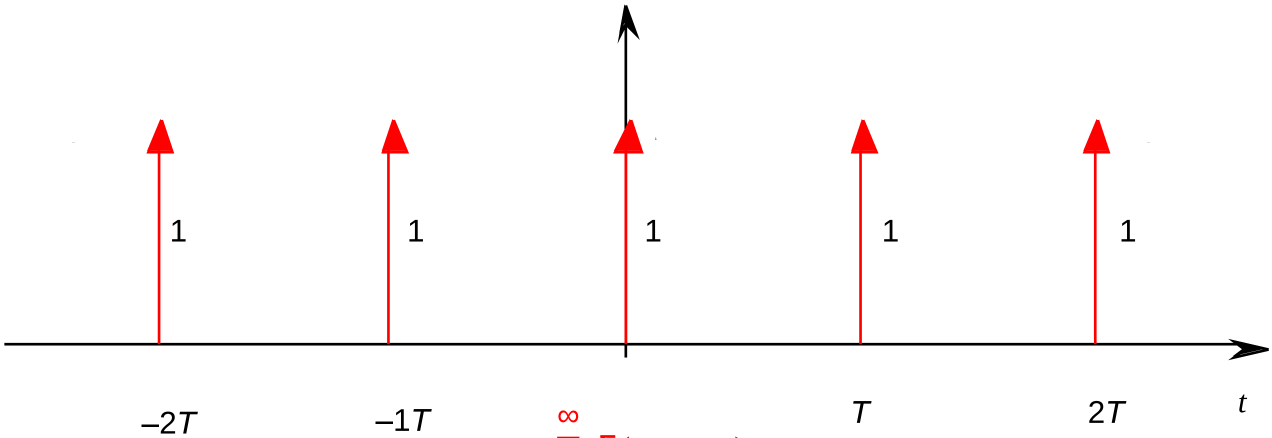
$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Egenskaper hos fouriertransformen

Funktion	Transform
Om $x(t)$	$Z(\omega)$
så $Z(t)$	$2\pi \cdot x(-\omega)$
$x(t)$	$X(\omega)$
$e^{i\omega_0 t}x(t)$	$X(\omega - \omega_0)$
$x(t - t_0)$	$e^{-i\omega t_0} X(\omega)$
$x(at), a \neq 0$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
$x(-t)$	$X(-\omega)$
$(x * y)(t)$	$X(\omega) \cdot Y(\omega)$
$x(t) \cdot y(t)$	$\frac{1}{2\pi}(X * Y)(\omega)$
$\frac{d}{dt} x(t)$	$i\omega X(\omega)$
$t\, x(t)$	$i \frac{d}{d\omega} X(\omega)$
$\left(\frac{d}{dt}\right)^n x(t)$	$(i\omega)^n X(\omega)$
$t^n x(t)$	$i^n \frac{d^n}{d\omega^n} X(\omega)$
Sampling av $x(t)$ med sampelavstånd T	$2\pi/T$-periodisk fortsättning av $1/T \cdot X(\omega)$
L-periodisk fortsättning av $x(t)$	Sampling av $2\pi/L \cdot X(\omega)$ med avstånd $2\pi/L$

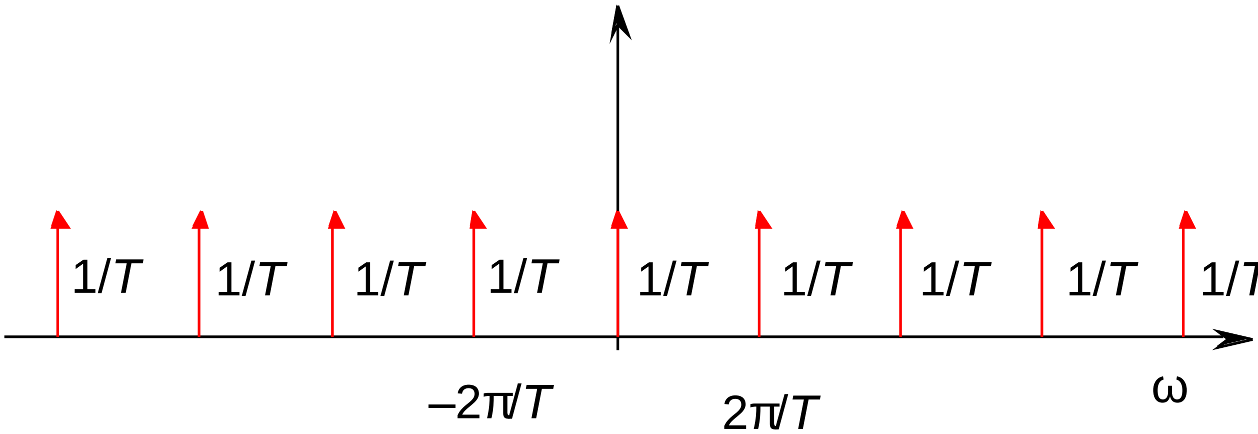
Speciella transformer

Funktion	Transform
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$\delta(t-t_0)$	$e^{-i\omega t_0}$
$e^{i\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$
$\delta(t-t_0)+\delta(t+t_0)$	$e^{-i\omega t_0}+e^{i\omega t_0}=2\cos(\omega t_0)$
$\cos(\omega_0 t)$	$\pi\,(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$
$\delta(t+t_0)-\delta(t-t_0)$	$e^{i\omega t_0}-e^{-i\omega t_0}=2i\sin(\omega t_0)$
$\sin(\omega_0 t)$	$-\pi i\,(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$2\pi/T\sum_{n=-\infty}^{\infty}\delta(\omega-2\pi n/T)$
$u(t)$	$\frac{1}{i\omega}+\pi\,\delta(\omega)$
$\operatorname{sign}(t)$	$\frac{2}{i\omega}$
$\operatorname{rect}(t/P)$	$P\operatorname{sinc}(P\omega/(2\pi))$
$\operatorname{sinc}(t/(2\pi))$	$2\pi\operatorname{rect}(\omega)$
$\operatorname{sinc}(t)$	$\operatorname{rect}(\omega/(2\pi))$
$u(t)$	$\frac{1}{i\omega}+\pi\,\delta(\omega)$
$\operatorname{sign}(t)$	$\frac{2}{i\omega}$



$$\sum_{n=-\infty}^{\infty} \delta(t - nT)$$

har fouriertransform



$$1/T \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n/T)$$