1010.

$$A = \oint_{\Gamma} (x+y)^2\,dx + (x-y)^2\,dy$$

Γ : $\begin{cases} \text{Ett varv i positiv led runt } \left(\frac{1}{2}, \frac{1}{2}\right) \\ \text{längs kurvorna } y = x^2 \text{ och } y^2 = x. \end{cases}$

$$\Gamma_1 : \left\{ y = x^2 \ , \ dy = 2x\,dx \ , \ x : 0 \to 1 \right.$$

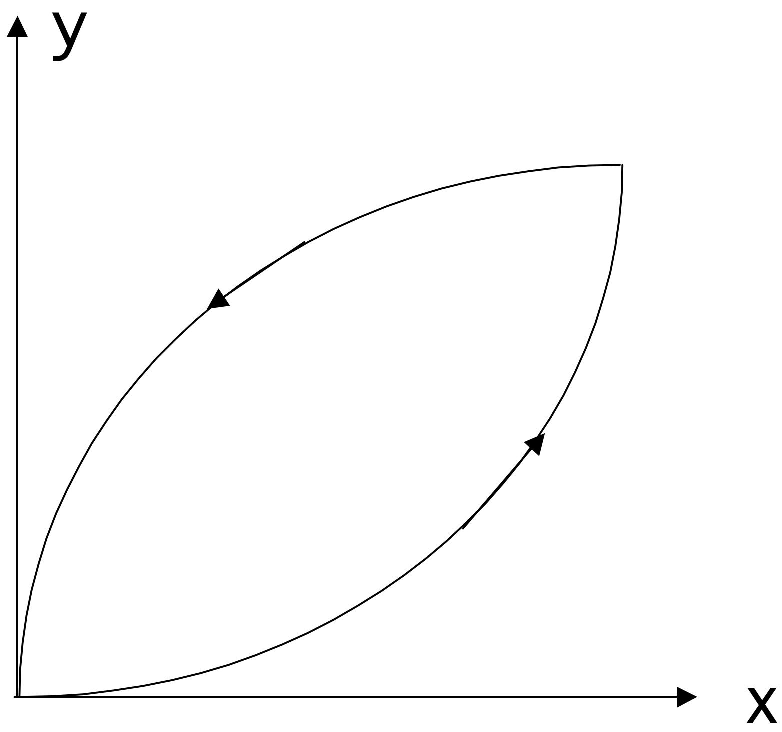
$$\Gamma_2 : \left\{ x = y^2 \ , \ dx = 2y\,dy \ , \ y : 1 \to 0 \right.$$

$$A = \int_{\square_1\,\square\square_2} (x+y)^2\,dx + (x-y)^2\,dy$$

$$A_1 = \int_{x=0}^{1} \left\{ (x + x^2)^2 + (x - x^2)^2\, 2x \right\} dx$$

$$A_2 = \int_{y=1}^{0} \left\{ (\mathrm{y}^2 + y)^2\, 2y + (\mathrm{y}^2 - y)^2 \right\} dy$$

$$A = A_1 + A_2 = \int_{y=0}^{1} \left\{ 4\mathrm{y}^3 - 8\mathrm{y}^4 \right\} dy = 1 - \frac{8}{5} = -\frac{3}{5}$$


y
x

$$A = \{\text{Greens formel}\} = \iint_D \left[\frac{\partial}{\partial x}(x - y)^2 - \frac{\partial}{\partial y}(x + y)^2 \right] dxdy$$

$$A = \iint_D -4y\,dxdy = -4\int_{y=0}^{1} \left(\int_{x=y^2}^{\sqrt{y}} y\,dx \right) dy = -4\int_{y=0}^{1} \left(y^{3/2} - y^3 \right) dy$$

$$A = -4\left(\frac{1}{5/2} - \frac{1}{4} \right) = 1 - \frac{8}{5} = -\frac{3}{5}$$