

1027.

$$A = \int_C (x dx + y dy) \arctan \frac{y}{x}$$

C : { Rätta linjen $(1, 0) \rightarrow (2, 2)$.

Slut med rätta linjen från $(1, 0)$ till $(2, 0)$, kurvan A ,
och vidare rätlinjigt till $(2, 2)$, kurvan B .

Inneslutna området betecknas D .

Greens formel tillämpas.

$$\int_A^B (x dx + y dy) \arctan \frac{y}{x} =$$

$$= \int_D \left[\frac{\partial}{\partial x} \left(y \arctan \frac{y}{x} \right) - \frac{\partial}{\partial y} \left(x \arctan \frac{y}{x} \right) \right] dx dy =$$

$$= \int_D \left[y \frac{-y}{x^2} - x \frac{1}{1 + \frac{y^2}{x^2}} \right] dx dy$$

$$\int_A^B (x dx + y dy) \arctan \frac{y}{x} = \int_D 1 dx dy$$

$$\int_A^B (x dx + y dy) \arctan \frac{y}{x} = \frac{1 \cdot 2}{2} = 1$$

$$\int_A (x dx + y dy) \arctan \frac{y}{x} = 0$$

$$\int_B (x dx + y dy) \arctan \frac{y}{x} = \int_{t=0}^2 t \arctan \frac{t}{2} dt$$

$$\int_B (x dx + y dy) \arctan \frac{y}{x} = \int_{t=0}^2 \frac{t^2}{2} \arctan \frac{t}{2} + \int_{t=0}^2 \frac{t^2}{2} \frac{1}{1 + \frac{t^2}{4}} dt$$

$$\int_B (x dx + y dy) \arctan \frac{y}{x} = 2 \frac{\square}{4} \int_{t=0}^2 \frac{t^2}{4 + t^2} dt$$

$$\int_B (x dx + y dy) \arctan \frac{y}{x} = \frac{1}{2} \int_0^2 2 + \frac{1}{2} \int_0^2 2 \arctan \frac{t^2}{2} dt = 0$$

$$\int_B (x dx + y dy) \arctan \frac{y}{x} = 0 \int_0^2$$

$$\int_0^1 1 + \int_A + \int_B = 1$$

$$\int_0^1 (x dx + y dy) \arctan \frac{y}{x} = 0 + 1 \int_0^2 + 1 = 1 \int_0^1$$