1041.

$$A = \oint_{\partial D} (y - 2x^2 y)\,dx + (3x - xy^2)\,dy$$

$$A = \iint_D \left(\frac{\partial}{\partial x}(-3x + xy^2) - \frac{\partial}{\partial y}(y - 2x^2 y) \right) dx\,dy$$

$$A = \iint_D (2x^2 + y^2 - 4)\,dx\,dy$$

A blir så liten som möjligt då $2x^2 + y^2 - 4 \le 0$.

$$\square: \; 2x^2 + y^2 = 4, \;\; \frac{x^2}{2} + \frac{y^2}{4} = 1$$

Inför elliptiska koordinater : $\begin{cases} \dfrac{x}{\sqrt{2}} = r\cos\square \\ \dfrac{y}{2} = r\sin\square \end{cases}$

$$dxdy = 2\sqrt{2}\,r\,dr\,d\square$$

$$A = \iint_{D_{r\square}} (2\cdot 2r^2\cos^2\square + 4r^2\sin^2\square \square 4)2\sqrt{2}\,r\,dr\,d\square$$

$$A = 8\sqrt{2} \cdot 2\pi \int_{r=0}^{1} (r^3 - r)dr = 16\pi\sqrt{2}\left(\frac{1}{4} - \frac{1}{2}\right) = -4\pi\sqrt{2}$$

$-4\pi\sqrt{2}$ 是 A 的