11.1.a.

$$\mathbf{F} = (x, \square y, z)$$

$$\square = \{(x,y,z): x + 2y + 3z = 1, x \geq 0, y \geq 0, z \geq 0\}$$

En normal $\mathbf{n} = grad(x + 2y + 3z \square 1) = (1,2,3)$.

$$\mathbf{F} \cdot \mathbf{n} = (x, \square y, z) \cdot (1,2,3) = x \square 2y + 3z$$

$$\mathbf{F} \cdot \mathbf{n} = \{\text{På } \square \text{ är } x + 2y + 3z = 1\} = 1 \square 4y$$

$$d\square = \frac{dxdy}{|\cos \square|} = \frac{dxdy}{|3/|\mathbf{n}||} = \frac{\sqrt{1^2 + 2^2 + 3^2}}{3} dxdy$$

$$\mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \frac{1-4y}{\sqrt{1^2+2^2+3^2}}\,\frac{\sqrt{1^2+2^2+3^2}}{3}\,dxdy$$

Flödet i normalriktningen genom σ är : $\iint_{\sigma}\mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma$.

$$\iint_{\sigma}\mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \frac{1}{3}\iint_{D}(1-4y)\,dxdy$$

$$\iint_{\sigma}\mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \frac{1}{3}\int_{y=0}^{1/2}\left(\int_{x=0}^{1-2y}(1-4y)\,dx\right)dy = \frac{1}{3}\int_{y=0}^{1/2}(1-4y)(1-2y)\,dy$$

$$\iint_{\sigma}\mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \frac{1}{3}\left[y+\frac{8}{3}y^3-3y^2\right]_{y=0}^{1/2} = \frac{1}{3}\left(\frac{1}{2}+\frac{1}{3}-\frac{3}{4}\right) = \frac{1}{36}$$

11.1.g.

$$\mathbf{F} = (y, x, z)$$

$$\iint_{S} \mathbf{F} \cdot \hat{\mathbf{n}}\, d\sigma$$

$$S: \begin{cases} x = u + v \\ y = u - v \\ z = u^2 \end{cases} \qquad |u| \le 1,\ |v| \le 1,\ n_z < 0$$

$$\begin{cases} \mathbf{r}_{\mathbf{u}} = (1,\ 1,\ 2u) \\ \mathbf{r}_v = (1,\ -1,\ 0) \end{cases}$$

$$\mathbf{n} = \mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ 1 & 1 & 2u \\ 1 & -1 & 0 \end{vmatrix} = (2u, +2u, -2)$$

$$\mathbf{n}_z < 0$$

$$\mathbf{F} = (u - v, u + v, u^2)$$

$$\mathbf{F} \cdot \mathbf{n} = (u - v, u + v, u^2) \cdot (2u, +2u, -2) = 2u^2$$

$$\iint_{\Sigma} \mathbf{F}\cdot\hat{\mathbf{n}}\, d\sigma = \iint_{D_{uv}} 2u^2\,du\,dv$$

$$D_{uv} = \left\{(u,v):\ |u| \le 1\ ,\ |v| \le 1\right\}$$

$$\iint_{\Sigma} \mathbf{F}\cdot\hat{\mathbf{n}}\, d\sigma = 2\cdot 2\cdot 2\cdot \frac{1}{3} = \frac{8}{3}$$

11.1.i.

$$\mathbf{F} = |\mathbf{r}|^{-3}\,\mathbf{r}\,, \quad \mathbf{r} = (x, y, z)$$

$\square$ är sfär med radien R och centrum i origo .

$\hat{\mathbf{n}}$ är utåtriktad .

$$\hat{\mathbf{n}} = \frac{\mathbf{r}}{R}$$

$$\mathbf{F}\cdot\hat{\mathbf{n}} = \frac{\mathbf{r}}{|\mathbf{r}|^3}\cdot\frac{\mathbf{r}}{R} = \frac{1}{rR} = \left\{\text{På } \square \text{ är } r = R.\right\} = \frac{1}{R^2}$$

$$\oiint_{\square} \mathbf{F}\cdot\hat{\mathbf{n}}\,d\square = \oiint_{\square} \frac{1}{R^2}\,d\square = \frac{1}{R^2}\cdot\text{Sfärens area} = \frac{1}{R^2}4\pi R^2 = 4\pi$$