11.5.d.

Visa att :

$$\iiint_K ((\mathrm{grad}\,u)\cdot \mathbf{F})\,dxdydz = \oiint_{\partial K} (u\mathbf{F})\cdot \hat{\mathbf{n}}\,d\sigma - \iiint_K (u\,\mathrm{div}\,\mathbf{F})\,dxdydz$$

$$\oiint_{\partial K} (u\mathbf{F})\cdot \hat{\mathbf{n}}\,d\sigma = \{\text{Divergenssatsen}\} = \iiint_K div\,(u\mathbf{F})\,dxdydz$$

$$div\,(u\mathbf{F}) = grad u\cdot \mathbf{F} + u\,div\,\mathbf{F}$$

$$\oiint_{\partial K} (u\mathbf{F})\cdot \hat{\mathbf{n}}\,d\sigma = \iiint_K (grad u\cdot \mathbf{F})\,dxdydz + \iiint_K (u\,div\,\mathbf{F})\,dxdydz$$

$$\oiint_{\partial K} (u\mathbf{F})\cdot \hat{\mathbf{n}}\,d\sigma - \iiint_K (u\,div\,\mathbf{F})\,dxdydz = \iiint_K (grad u\cdot \mathbf{F})\,dxdydz$$

$$div\,(u\mathbf{F}) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(u\mathbf{F}_1, u\mathbf{F}_2, u\mathbf{F}_3)$$

$$div\,(u\mathbf{F}) = \frac{\partial u}{\partial x}\mathbf{F}_1 + u\frac{\partial \mathbf{F}_1}{\partial x} + \frac{\partial u}{\partial y}\mathbf{F}_2 + u\frac{\partial \mathbf{F}_2}{\partial y} + \frac{\partial u}{\partial z}\mathbf{F}_3 + u\frac{\partial \mathbf{F}_3}{\partial z}$$

$$div\,(u\mathbf{F}) = grad\,u \cdot \mathbf{F} + u\,div\,\mathbf{F}$$