919.a.

$$V = \iint_D \sqrt{x^2 + y^2}\, dxdy$$

$$D = \left\{(x,y): x^2 + y^2 \le 1,\ x \ge 0,\ y \ge 0\right\}$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \qquad dxdy = rdrd\varphi$$

$$D_{r\varphi} = \left\{(r,\varphi): 0 \le r \le 1,\ 0 \le \varphi \le \frac{\pi}{2}\right\}$$

$$V = \iint_{D_{r\varphi}} r\, rdrd\varphi = \frac{\pi}{2}\int_{r=0}^{1} r^2 dr = \frac{\pi}{2}\frac{1}{3} = \frac{\pi}{6}$$

919.b.

$$D = \left\{(x,y): x^2 + y^2 \leq y,\ x \geq 0\right\}$$

$$D = \left\{(x,y): x^2 + \left(y - \frac{1}{2}\right)^2 \leq \left(\frac{1}{2}\right)^2,\ x \geq 0\right\}$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \qquad dxdy = rdrd\varphi$$

$$D_{r\varphi} = \left\{(r,\varphi): 0 \leq r \leq \sin\varphi,\ 0 \leq \varphi \leq \frac{\pi}{2}\right\}$$

$$V = \iiint_{D_{r\varphi}} r\,r\,dr\,d\varphi\,d\theta = \int_{\varphi=0}^{\frac{\pi}{2}} \left(\int_{r=0}^{\sin\varphi} r^2\,dr \right) d\varphi$$

$$V = \frac{1}{3} \int_{\varphi=0}^{\frac{\pi}{2}} \sin^3\varphi\,d\varphi = \{\text{BETA } 7.5.17.\} = \frac{1}{3}\frac{2!!}{3!!}$$

$$V = \{\text{BETA } 2.1.\} = \frac{1}{3}\frac{2}{3} = \frac{2}{9}$$

$$V = \left| \begin{array}{l} \sin^3\varphi = (1-\cos^2\varphi)\sin\varphi \\ t = \cos\varphi,\ \ dt = -\sin\varphi\,d\varphi \end{array} \right| = \frac{1}{3} \int_{t=1}^{0} -(1-t^2)\,dt$$

919.c.

$$D = \{(x,y): y \le x^2 + y^2 \le 2y\}$$

$$D = \left\{(x,y): x^2 + (y - \frac{1}{2})^2 \ge \left(\frac{1}{2}\right)^2,\ x^2 + (y-1)^2 \le 1^2\right\}$$

$$D_{r\varphi} = \{(r,\varphi): \sin\varphi \le r \le 2\sin\varphi,\ 0 \le \varphi \le \pi\}$$

$$V = \iint_{D_{r\varphi}} r\,r\,dr\,d\varphi = \int_{\varphi=0}^{\pi} \left(\int_{r=\sin\varphi}^{2\sin\varphi} r^2 dr \right) d\varphi$$

$$V = \left| \begin{array}{l} \sin^3\varphi = (1-\cos^2\varphi)\sin\varphi \\ t = \cos\varphi,\ \ dt = -\sin\varphi\, d\varphi \end{array} \right| = \frac{7}{3} \int_{t=1}^{-1} (1-t^2)dt = 2\frac{7}{3}\frac{2}{3} = \frac{28}{9}$$

919.d.

$$D = \left\{(x,y): y \le x^2 + y^2 \le x\right\}$$

$$D = \left\{(x,y): x^2 + (y - \frac{1}{2})^2 \ge \left(\frac{1}{2}\right)^2,\ (x - \frac{1}{2})^2 + y^2 \le \left(\frac{1}{2}\right)^2\right\}$$

$$D_{r\varphi} = D_1 \cup D_2 \quad \begin{cases} D_1 = \left\{(r,\varphi): 0 \le r \le \cos\varphi,\ -\frac{\pi}{2} \le \varphi \le 0\right\} \\ D_2 = \left\{(r,\varphi): \sin\varphi \le r \le \cos\varphi,\ 0 \le \varphi \le \frac{\pi}{4}\right\} \end{cases}$$

$$V = \iint_{D_{r\varphi}} r\,dr\,d\varphi = \int_{\varphi=-\frac{\pi}{2}}^{0}\left(\int_{r=0}^{\cos\varphi} r^2\,dr\right)d\varphi + \int_{\varphi=0}^{\frac{\pi}{4}}\left(\int_{r=\sin\varphi}^{\cos\varphi} r^2\,dr\right)d\varphi$$

$$V = \int_{\varphi=-\frac{\pi}{2}}^{0} \frac{\cos^3\varphi}{3}\,d\varphi + \int_{\varphi=0}^{\frac{\pi}{4}} \frac{\cos^3\varphi\,\sin^3\varphi}{3}\,d\varphi$$

BETA 7.4.207 - 208.

$$V = \frac{1}{3}\left[\sin\varphi\right]_{\varphi=-\frac{\pi}{2}}^{0} - \frac{1}{3}\sin^3\varphi\Big|_{\varphi=-\frac{\pi}{2}}^{0} \int_{\varphi=0}^{\frac{\pi}{4}} \cos\varphi + \frac{1}{3}\cos^3\varphi\Big|_{\varphi=0}^{\frac{\pi}{4}}$$

$$V = \frac{1}{3}\left[1 - \frac{1}{\sqrt{2}}\right] - \frac{1}{3}\frac{1}{2\sqrt{2}} + 1 - \frac{1}{3}\left[\frac{1}{\sqrt{2}} + \frac{1}{3}\frac{1}{2\sqrt{2}} + 1 - \frac{1}{3}\right]$$

$$V = \frac{1}{3}\frac{1}{\sqrt{2}}\,2\,\frac{5}{6} = \frac{5\sqrt{2}}{18}$$