937.l.

$$V = \iint_D \frac{dxdy}{(x^2+4y^2)^2}$$

$$D = \{(x,y) : x > 0 , \ xy > 1\}$$

$$\begin{cases} x = 2r\cos v \\ y = r\sin v \end{cases} \qquad \frac{d(x,y)}{d(r,v)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2\cos v & -2r\sin v \\ \sin v & r\cos v \end{vmatrix}$$

$$dxdy = \left| \det\left(\frac{d(x,y)}{d(u,v)}\right) \right| dudv = 2rdrdv$$

$$D_{rv} = \left\{(r,v) : r > \frac{1}{\sqrt{\sin 2v}} , \ 0 < v < \frac{\pi}{2}\right\}$$

$$V = \iiint_{D_{rv}} \frac{2r\,dr\,dv}{16r^4} = \frac{1}{16}\int_{v=0}^{\frac{\pi}{2}} \left(\int_{r=\frac{1}{\sqrt{\sin 2v}}} \frac{2dr}{r^3} \right) dv$$

$$V = \frac{1}{16}\int_{v=0}^{\frac{\pi}{2}} \left[-\frac{1}{r^2} \right]_{r=\frac{1}{\sqrt{\sin 2v}}} dv = \frac{1}{16}\int_{v=0}^{\frac{\pi}{2}} \sin 2v\,dv$$

$$V = \frac{1}{16}\left[-\frac{\cos 2v}{2} \right]_{v=0}^{\frac{\pi}{2}} = \frac{1}{16}$$

937.r.

$$M = \iiint_K \left(\frac{1}{\sqrt{x^2 + y^2}} + \frac{1}{z} \right) dxdydz$$

$$K = \left\{ (x,y,z) : \; z > 0 \; , \; x^2 + y^2 + z^2 \leq 1 \; , \; x^2 + y^2 < z^2 \right\}$$

$$\begin{cases} x = r \sin\varphi \cos\theta \\ y = r \sin\varphi \sin\theta \\ z = r \cos\varphi \end{cases}$$

$$dxdydz = r^2 \sin\varphi \, dr \, d\varphi \, d\theta$$

$$K_{r\varphi\theta} = \left\{ (r,\varphi,\theta) : \; 0 \leq r \leq 1 \; , \; 0 \leq \varphi \leq \frac{\pi}{4} \; , \; 0 \leq \theta < 2\pi \right\}$$

$$M = \iiint_{K_{r\theta\varphi}} \left(\frac{1}{r\sin\varphi} + \frac{1}{r\cos\varphi}\right) r^2 \sin\varphi \, dr\, d\varphi\, d\theta$$

$$M = 2\pi \iint_{D_{r\varphi}} \left(1 + \frac{\sin\varphi}{\cos\varphi}\right) r\, dr\, d\varphi$$

$$M = \pi\left[\left(\varphi - \ln(\cos\varphi)\right)\right]_{\varphi=0}^{\frac{\pi}{4}} = \pi\left(\frac{\pi}{4} - \ln\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}(\pi + \ln 4)$$