

LS1, Version A.

Beräkna dubbelintegralen

$$I = \iint_D (x^2 + y^2) dx dy$$

$$D = \left\{ (x, y) : 4 \leq x^2 + y^2 \leq 9, 0 \leq x \leq y \right\}$$

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

$$dx dy = r dr d\theta$$

$$D_{r\theta} = \left\{ (r, \theta) : 2 \leq r \leq 3, \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4} \right\}$$

$$I = \iint_{D_{r\theta}} r^2 r dr d\theta$$

$$I = \left(\frac{\pi}{2} - \frac{\pi}{4} \right) \int_{r=2}^3 r^3 dr = \frac{\pi}{4} \frac{3^4 - 2^4}{4} = \frac{\pi}{4} \frac{81 - 16}{4} = \frac{65\pi}{16}$$