LS6. Version A.

$$\dot{\mathbf{X}} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\mathbf{X} + \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$$

$$\dot{\mathbf{X}} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\mathbf{X}$$

$$0 = \begin{vmatrix} 3-\lambda & -2 \\ 1 & 0-\lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2)$$

$$\lambda_1 = 1 : \quad \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\mathbf{K} = \mathbf{0}\,, \quad \mathbf{K_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad \mathbf{X_1} = e^{t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\lambda_2 = 2: \quad \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\mathbf{K} = \mathbf{0}, \quad \mathbf{K}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \quad \mathbf{X}_2 = e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

$$\mathbf{X}_h = c_1\mathbf{X}_1 + c_2\mathbf{X}_2 = c_1\begin{pmatrix} e^{t} \\ e^{t} \end{pmatrix} + c_2\begin{pmatrix} 2e^{2t} \\ e^{2t} \end{pmatrix} = \begin{pmatrix} e^{t} & 2e^{2t} \\ e^{t} & e^{2t} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\mathbf{X}_p = \Phi \int \Phi^{-1}\begin{pmatrix} e^{2t} \\ -e^{2t} \end{pmatrix} dt$$

$$\Phi = \begin{pmatrix} e^{t} & 2e^{2t} \\ e^{t} & e^{2t} \end{pmatrix}$$

$$\Phi^{-1} = \frac{1}{-e^{3t}}\begin{pmatrix} e^{2t} & -2e^{2t} \\ -e^{t} & e^{t} \end{pmatrix} = \begin{pmatrix} -e^{-t} & 2e^{-t} \\ e^{-2t} & -e^{-2t} \end{pmatrix}$$

$$\int \begin{pmatrix} -e^{-t} & 2e^{-t} \\ e^{-2t} & -e^{-2t} \end{pmatrix} \begin{pmatrix} e^{2t} \\ -e^{2t} \end{pmatrix} dt = \int \begin{pmatrix} -3e^{t} \\ 2 \end{pmatrix} dt = \begin{pmatrix} -3e^{t} \\ 2t \end{pmatrix}$$

$$\mathbf{X}_p = \begin{pmatrix} e^{t} & 2e^{2t} \\ e^{t} & e^{2t} \end{pmatrix} \begin{pmatrix} -3e^{t} \\ 2t \end{pmatrix} = \begin{pmatrix} -3e^{2t} + 4te^{2t} \\ -3e^{2t} + 2te^{2t} \end{pmatrix} = e^{2t} \begin{pmatrix} 4t - 3 \\ 2t - 3 \end{pmatrix}$$

$$\mathbf{X} = \mathbf{X}_h + \mathbf{X}_p = \begin{pmatrix} e^{t} & 2e^{2t} \\ e^{t} & e^{2t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + e^{2t} \begin{pmatrix} 4t - 3 \\ 2t - 3 \end{pmatrix}$$