KTH Matematik

Interplay between systems and linear functions
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Recall that given a linear function $A : \mathbb{R}^n \to \mathbb{R}^m$ the *range* , of A is defined as:

$R(A) = \{\vec{b} \in \mathbb{R}^m \text{ such that there is } \vec{x} \in \mathbb{R}^n \text{ with } A(\vec{x}) = \vec{b}\}$.

Then using the system notation it is:

$\mathbf{R}(\mathbf{A}) = \{\vec{b} \in \mathbb{R}^m \text{ such that there is } \vec{x} \in \mathbb{R}^n \text{ with } A(\vec{x}) = \vec{b}\} \subseteq \mathbb{R}^m$.

From what we have seen last time it follows that:

$$\mathbf{R}(\mathbf{A}) = \mathbf{Col}(\mathbf{A})$$

Let $\mathbf{Sol_b}(\mathbf{A}) = \{\vec{v}, \text{ solution for } A\vec{v} = \vec{b}\}$.

Then there is the following correspondence:

- $N(A) = Sol_0(F)$
- $\vec{b} \in R(A) \Leftrightarrow Sol_b(A) \neq \emptyset$

Using this terminology we drow the following conclusions:

$$N(A) = Sol_0(F) = \{\vec{0}\} \quad \Leftrightarrow \quad A : \mathbb{R}^n \to \mathbb{R}^m \text{ is one-to-one}$$

$$Sol_b(A) \neq \emptyset \text{ for all } \vec{b} \in \mathbb{R}^m \quad \Leftrightarrow \quad R(A) = \mathbb{R}^m$$

$$n = rk(A) + nullity(A) \quad \Leftrightarrow \quad n = \dim(R(A)) + \dim(N(A))$$