5B1308 Wavelets

Inlämningsuppgift nr 4

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Assignment IV for Wavelet Course 2005 fall

Handed out:November 1,
To be handed in before December 4.

Part A Construction of Daubechies Orthogonal wavelet filter of length 6.

1. Write Z transform of lowpass filter h as $H(z) = (1 + \frac{1}{z})^3 P(\frac{1}{z})$ where P is a polynom of degree 2. It remains to find P.
2. Let $Q(z) = P(z)P(\frac{1}{z})$.
3. let q and b be the filters with Z transforms $Q(z)$ and $(1 + z)^3(1 + \frac{1}{z})^3$
4. Find q so that $\{T^{2k}q\}_k$ and $\{T^{2k}b\}_k$ will be bi-orthogonal filter by solving a few linear equations. Use this to find Q
5. Find the polynomial P from knowing Q.
   (This is numerically the hardest part)
6. Finally find from this the lowpass filter h and the highpassfilter g

Part B The local cosine transform
This assignment is rather new for me too, so I dont know how it will turn out. If you get problems with it dont hesitate to contact me
/Jan-Olov S

The data consists of N sample points. We assume the sampling is done at half points between integers: $n + \frac{1}{2}$. The sampling intervall is broken up into intervals $[a_{j-1}, a_j]$ where a_j are integers.
Around each intervall endpoint a_j, $0 < j < 2^k$ there is a transition interval $[a_j - b_j, a_j + b_j]$ where windows overlapp. Here b_j is an integer and $b_j + b_{j-1} \leq a_j - a_{j-1}$ so that the transision intervals do not overlapp. (For simplicity we may assume that $N = 2^m$ and there are 2^k intervals of lengths $L_j = a_j - a_{j-1} = L = 2^{(m-k)}$ and where $0 < k < m$ and also that all b_j are equal: $b_j = b$)

1. Build a folding matrix (and its inverse: the unfolding matrix).
   This will be a matrix $\{c_{jk}\}$ of size $2b \times 2b$ with non-zero entries only on two crossing diagonals. There values are given by the 2×2 rotation matrices

   $$\begin{pmatrix} c_{j,j} & c_{j,2b-j+1} \\ c_{2b-j+1,j} & c_{2b-j+1,2b+j-1} \end{pmatrix} = \begin{pmatrix} \cos\alpha_j & -\sin\alpha_j \\ \sin\alpha_j & \cos\alpha_j \end{pmatrix} \text{ for } j = 1, \ldots, b,$$

   where the rotation angles

   $$\alpha_j = \frac{\pi}{4} P\left(\frac{j - b - \frac{1}{2}}{b}\right) \text{ for } j = 1, \ldots, b.$$

   and $P(t)$ is a function as in the construction of the Daubechies filter(with properties $0 \leq P(t) \leq 2$, $P(t)+P(-t) = 2$. $P(t)$ and some of its derivatives are zero at $t = -1$.

2. Folding the data in all the transition intervalls. The matrix above applies to the data in the transition interval for each transition interval. Use this matrix to modify those data points wich belongs to these transition intervals.

3. After this may orginize the folded data into with 2^k collumns of length 2^{m-k} using reshape.

4. Next we want to apply to each collumnvector the inner product with $\cos(k + \frac{1}{2})t/L$
   Extend for each j the jth column to double length by odd reflection around highest index $+\frac{1}{2}$ to obtain data $\{f_j(n)\}_n$.

5. For each extended collumm we obtain the coeficients as $c_{kj} = \frac{1}{2}\{\hat{f}_j(k - \frac{1}{2}) + \hat{f}_j(-(k - \frac{1}{2}))\}$ for $k = 1 \ldots, L$
   . (if data is real take real part:$c_{kj} = \hat{f}(k - \frac{1}{2})$)
   Since sampling points are at half integer points and also we looking at half integer frequences

   $$\hat{f}_j(k - \frac{1}{2}) = \frac{1}{\sqrt{L}} \sum_{k=1}^{2L} f_j(n) \exp\{i\pi(k - \frac{1}{2})(n - \frac{1}{2})/L\}$$

6. Compare the formula above with matlabs fft by using 'help fft'. Find factors A_n and B_k so that the formulas will be consistent with $x(n) = A_n f(n)$ and $\hat{f}(k) = B_k X(k)$, where $x(n)$ and Xk are given by the formula in 'help matlab'

7. Try to Inverse the process seting

   $$\hat{f}(k - \frac{1}{2}) = \hat{f}(-(k - \frac{1}{2})) = c_{kj}$$

take inverse fft with matlab involving the weight factors A_n and B_k.Now you should have a matrix consisting of $2^{m=k}$ columns of length $2L$
Restrict the columns to length L and reshape the matrix to get a vector of length N.
Finally use the unfolding matrix to modify the data at the transition intervals.

8. Try the transform on the data guitar.wav. You may choose a subseequence of suitable length. Suggestion set $b = 8$ and $L = 64$.