5B1473 Elementary differential geometry, spring 2007

Homework problems 5 — To be handed in 29/5

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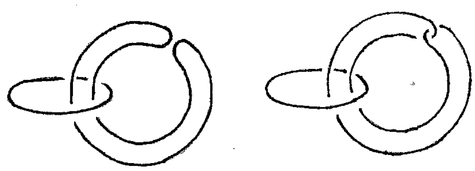
1 Show that every map $S^n \to T^n = S^1 \times \cdots \times S^1$ has degree zero.
(*hint:* The volume form on T^n is a product.)

2 Compute the linking number of the following pairs of curves:



3 Sketch vector fields on S^2 and T^2 having only isolated singular points, at least one of which has index 3.

4 Let M^2 and N^0 be submanifolds of $\mathbb{R}^3$, where M^2 is diffeomorphic to S^2 and N^0 is a point. Compute the linking number $lk(M,N)$. What is the result if M^2 is any compact oriented 2-dim. manifold?

5 Suppose that M, N are compact manifolds without boundary with Morse functions f, g respectively. Show that $h = f + g$ is a Morse function on $M \times N$ and use this to show that $\chi(M \times N) = \chi(M)\chi(N)$.

6 Let M be a compact manifold without boundary and let $\pi : \widetilde{M} \to M$ be a d-fold covering. That is, M can be covered with open sets U with the property that $\pi^{-1}(U)$ is disjoint union of d open sets $U_1, \ldots, U_d$ such that $\pi_{|U_i} : U_i \to U$ is a diffeomorphism. Show that $\chi(\widetilde{M}) = d\chi(M)$.

7 Find a Morse function on a compact oriented two-dimensional manifold M with g holes and use this to compute the Euler number.