

Några lösta uppgifter ur kapitel 4

4.2.4 Visa $A \rightarrow B \vdash A \& B \rightarrow B \& C$

1	(1)	$A \rightarrow B$	<i>Premiss</i>
2	(2)	$A \& C$	<i>Antagande</i>
2	(3)	A	2 &E
1, 2	(4)	B	1, 3 \rightarrow E
2	(5)	C	2 &E
1, 2	(6)	$B \& C$	4, 5 &I
1	(7)	$A \& C \rightarrow B \& C$	2, 6 \rightarrow I

4.2.8 Visa $A \rightarrow B \vdash (C \rightarrow A) \rightarrow (C \rightarrow B)$

1	(1)	$A \rightarrow B$	<i>Premiss</i>
2	(2)	$C \rightarrow A$	<i>Antagande</i>
3	(3)	C	<i>Antagande</i>
2, 3	(4)	A	2, 3 \rightarrow E
1, 2, 3	(5)	B	1, 4 \rightarrow E
1, 2	(6)	$C \rightarrow B$	3, 5 \rightarrow I
1	(7)	$(C \rightarrow A) \rightarrow (C \rightarrow B)$	2, 6 \rightarrow I

4.2.12 Visa $C \rightarrow A, C \rightarrow (A \rightarrow B) \vdash C \rightarrow A \& B$

1	(1)	$C \rightarrow A$	<i>Premiss</i>
2	(2)	$C \rightarrow (A \rightarrow B)$	<i>Premiss</i>
3	(3)	C	<i>Antagande</i>
1, 3	(4)	A	1, 3 \rightarrow E
2, 3	(5)	$A \rightarrow B$	2, 3 \rightarrow E
1, 2, 3	(6)	B	4, 5 \rightarrow E
1, 2, 3	(7)	$A \& B$	4, 6 &I
1, 2	(8)	$C \rightarrow A \& B$	3, 7 \rightarrow I

4.3.1 Visa $\vdash A \& B \rightarrow B \& A$

1	(1)	$A \& B$	<i>Antagande</i>
1	(2)	A	1 &E
1	(3)	B	1 &E
1	(4)	$B \& A$	2, 3 &I
	(5)	$A \& B \rightarrow B \& A$	1, 4 \rightarrow I

4.3.2 Visa $\vdash (A \rightarrow B) \rightarrow (A \rightarrow B)$

1	(1)	$A \rightarrow B$	<i>Antagande</i>
2	(2)	A	<i>Antagande</i>
1, 2	(3)	B	1, 2 \rightarrow E
1	(4)	$A \rightarrow B$	2, 3 \rightarrow I
	(5)	$(A \rightarrow B) \rightarrow (A \rightarrow B)$	1, 4 \rightarrow I

4.4.6 Visa $\sim(A \& B), A \vdash \sim B$

1	(1)	$\sim(A \& B)$	<i>Premiss</i>
2	(2)	A	<i>Premiss</i>
3	(3)	B	<i>Antagande</i>
2, 3	(4)	$A \& B$	2, 3 &I
1, 2, 3	(5)	λ	1, 4 \sim E
1, 2	(6)	$\sim B$	3, 5 \sim I

4.4.15 Visa $A \rightarrow \sim(B \& C)$, $B \rightarrow C \vdash A \rightarrow \sim B$

1	(1)	$A \rightarrow \sim(B \& C)$	<i>Premiss</i>
2	(2)	$B \rightarrow C$	<i>Premiss</i>
3	(3)	A	<i>Antagande</i>
4	(4)		<i>Antagande</i>
2, 4	(5)		$2, 4 \rightarrow E$
2, 4	(6)		$4, 5 \& I$
1, 3	(7)		$1, 3 \rightarrow E$
1, 2, 3, 4	(8)		$6, 7 \sim E$
1, 2, 3	(9)	$\sim B$	$4, 8 \sim I$
1, 2	(10)	$A \rightarrow \sim B$	$3, 10 \rightarrow I$

4.4.22 Visa $\sim(A \rightarrow B) \vdash A \& \sim B$ och $A \& \sim B \vdash \sim(A \rightarrow B)$

1	(1)	$\sim(A \rightarrow B)$	<i>Premiss</i>
2	(2)	$\sim A$	<i>Antagande</i>
3	(3)		<i>Antagande</i>
4	(4)		<i>Antagande</i>
2, 3	(5)		$2, 3 \sim E$
2, 3	(6)		$4, 5 \sim I$
2, 3	(7)		6 DN
2	(8)	$A \rightarrow B$	$\rightarrow 3, 7$
1, 2	(9)	λ	$1, 8 \sim E$
1	(10)	$\sim \sim A$	$2, 9 \sim I$
1	(11)	A	10 DN
12	(12)	B	<i>Antagande</i>
13	(13)		<i>Antagande</i>
12	(14)		12
12	(15)		$13, 14 \rightarrow I$
1, 12	(16)	λ	$1, 15 \sim E$
1	(17)	$\sim B$	$12, 16 \sim I$
1	(18)	$A \& \sim B$	$11, 17 \& I$

1	(1)	$A \& \sim B$	<i>Premiss</i>
2	(2)	$A \rightarrow B$	<i>Antagande</i>
1, 2	(3)	A	$1 \& E$
1, 2	(4)	B	$1, 2 \rightarrow E$
1, 2	(5)	$\sim B$	$1 \& E$
1, 2	(6)	λ	$4, 5 \sim E$
1	(7)	$\sim(A \rightarrow B)$	$2, 6 \sim I$

4.5.8 Visa $A \vee B$, $A \rightarrow B$, $B \rightarrow A \vdash A \& B$

1	(1)	$A \vee B$	<i>Premiss</i>
2	(2)	$A \rightarrow B$	<i>Premiss</i>
3	(3)	$B \rightarrow A$	<i>Premiss</i>
4	(4)	A	<i>Antagande</i>
2, 4	(5)	B	$2, 4 \rightarrow E$
2, 4	(6)	$A \& B$	$4, 5 \& I$
7	(7)	B	<i>Antagande</i>
3, 7	(8)	A	$3, 7 \rightarrow E$
3, 7	(9)	$A \& B$	$7, 8 \& I$
1, 2, 3	(10)	$A \& B$	$1, 4, 6, 7, 9 \vee E$

4.5.14 Visa $A \& B \vdash \sim(\sim A \vee \sim B)$ och $\sim(\sim A \vee \sim B) \vdash A \& B$

1	(1)	$A \& B$	<i>Premiss</i>
2	(2)	[$\sim A \vee \sim B$	<i>Antagande</i>
1	(3)	A	1 &E
1	(4)	B	1 &E
5	(5)	[$\sim A$	<i>Antagande</i>
1, 5	(6)	λ	3,5 \sim E
7	(7)	[$\sim B$	<i>Antagande</i>
1, 7	(8)	λ	4,7 \sim E
1, 2	(9)	λ	2,5,6,7,8 \vee E
1	(10)	$\sim(\sim A \vee \sim B)$	2,9 \sim I

1	(1)	$\sim(\sim A \vee \sim B)$	<i>Premiss</i>
2	(2)	[$\sim A$	<i>Antagande</i>
2	(3)	$\sim A \vee \sim B$	2 \vee I
1, 2	(4)	λ	1,3 \sim E
1	(5)	$\sim\sim A$	2,4 \sim I
1	(6)	A	5 DN
7	(7)	[$\sim B$	<i>Antagande</i>
7	(8)	$\sim A \vee \sim B$	7 \vee I
1, 7	(9)	λ	1,8 \sim E
1	(10)	$\sim\sim B$	7,9 \sim I
1	(11)	B	10 DN
1	(12)	$A \& B$	6,11 &I

4.5.17 Visa $A \rightarrow B \vdash \sim A \vee B$ och $\sim A \vee B \vdash A \rightarrow B$.

1	(1)	$A \rightarrow B$	<i>Premiss</i>
2	(2)	[$\sim(\sim A \vee B)$	<i>Antagande</i>
3	(3)	[A	<i>Antagande</i>
1, 3	(4)	B	1,3 \rightarrow E
1, 3	(5)	$\sim A \vee B$	4 \vee I
1, 2, 3	(6)	λ	2,5 \sim E
1, 2	(7)	$\sim A$	3,6 \sim I
1, 2	(8)	$\sim A \vee B$	7 \vee I
1, 2	(9)	λ	2,8 \sim E
1	(10)	$\sim\sim(\sim A \vee B)$	2,9 \sim I
1	(11)	$\sim A \vee B$	10 DN

1	(1)	$\sim A \vee B$	<i>Premiss</i>
2	(2)	[A	<i>Antagande</i>
3	(3)	[$\sim B$	<i>Antagande</i>
4	(4)	[$\sim A$	<i>Antagande</i>
2, 4	(5)	λ	2,4 \sim E
6	(6)	[B	<i>Antagande</i>
3, 6	(7)	λ	3,6 \sim E
1, 2, 3	(8)	λ	1,4,5,6,7 \vee E
1, 2	(9)	$\sim\sim B$	3,8 \sim E
1, 2	(10)	B	9 DN
1	(11)	$A \rightarrow B$	2,10 \rightarrow I

4.5.18 Visa $\sim(A \rightarrow B) \vdash \sim(\sim A \vee B)$

1	(1)	$\sim(A \rightarrow B)$	<i>Premiss</i>
2	(2)	$\sim A \vee B$	<i>Antagande</i>
3	(3)	$\sim A$	<i>Antagande</i>
4	(4)	A	<i>Antagande</i>
5	(5)	$\sim B$	<i>Antagande</i>
3,4	(6)	λ	3,4 $\sim E$
3,4	(7)	$\sim \sim B$	5,6 $\sim I$
3,4	(8)	B	7 DN
3	(9)	$A \rightarrow B$	4,8 $\rightarrow I$
10	(10)	B	<i>Antagande</i>
11	(11)	A	<i>Antagande</i>
10	(12)	B	10
10	(13)	$A \rightarrow B$	11,12 $\rightarrow I$
2	(14)	$A \rightarrow B$	2,3,9,10,13 $\vee E$
1,2	(15)	λ	1,14 $\sim E$
1	(16)	$\sim(\sim A \vee B)$	2,15 $\sim I$

1	(1)	$\sim(\sim A \vee B)$	<i>Premiss</i>
2	(2)	$A \rightarrow B$	<i>Antagande</i>
3	(3)	B	<i>Antagande</i>
3	(4)	$\sim A \vee B$	3 $\vee I$
1,3	(5)	λ	1,4 $\sim E$
1	(6)	$\sim B$	3,5 $\sim I$
7	(7)	A	<i>Antagande</i>
2,7	(8)	B	2,7 $\rightarrow E$
1,2,7	(9)	λ	6,8 $\sim E$
1,2	(10)	$\sim A$	7,9 $\sim I$
1,2	(11)	$\sim A \vee B$	10 $\vee I$
1,2	(12)	λ	1,11 $\sim E$
1	(13)	$\sim(A \rightarrow B)$	2,12 $\sim I$

4.6.8 Visa $A \& B \vdash A \leftrightarrow B$

1	(1)	$A \& B$	<i>Premiss</i>
2	(2)	A	<i>Antagande</i>
1	(3)	B	1 $\&E$
1	(4)	$A \rightarrow B$	2,3 $\rightarrow I$
5	(5)	B	<i>Antagande</i>
1	(6)	A	1 $\&E$
1	(7)	$B \rightarrow A$	5,6 $\rightarrow I$
1	(8)	$(A \rightarrow B) \& (B \rightarrow A)$	4,7 $\&I$
1	(9)	$A \leftrightarrow B$	8 Df

4.6.14 Visa $\sim A \vee C, \sim B \vee \sim C \vdash A \rightarrow \sim(A \leftrightarrow B)$

1	(1)	$\sim A \vee C$	<i>Premiss</i>
2	(2)	$\sim B \vee \sim C$	<i>Premiss</i>
3	(3)	$\left[\begin{array}{l} A \\ \vdots \end{array} \right.$	<i>Antagande</i>
4	(4)	$\left[\begin{array}{l} A \leftrightarrow B \\ \vdots \end{array} \right.$	<i>Antagande</i>
4	(5)	$\left[\begin{array}{l} (A \rightarrow B) \& (B \rightarrow A) \\ \vdots \end{array} \right.$	4 Df
6	(6)	$\left[\begin{array}{l} \sim A \\ \vdots \end{array} \right.$	<i>Antagande</i>
7	(7)	$\left[\begin{array}{l} \sim C \\ \vdots \end{array} \right.$	<i>Antagande</i>
3, 6	(8)	$\left[\begin{array}{l} \lambda \end{array} \right.$	3,6 $\sim E$
3, 6	(9)	$\left[\begin{array}{l} C \\ \vdots \end{array} \right.$	7,8 $\sim I$
10	(10)	$\left[\begin{array}{l} C \\ \vdots \end{array} \right.$	<i>Antagande</i>
1, 3	(11)	$\left[\begin{array}{l} C \\ \vdots \end{array} \right.$	1,6,8,10 $\vee E$
12	(12)	$\left[\begin{array}{l} \sim B \\ \vdots \end{array} \right.$	<i>Antagande</i>
4	(13)	$\left[\begin{array}{l} A \rightarrow B \\ \vdots \end{array} \right.$	5 $\& E$
3, 4	(14)	$\left[\begin{array}{l} B \\ \vdots \end{array} \right.$	3,13 $\rightarrow E$
3, 4, 12	(15)	$\left[\begin{array}{l} \lambda \\ \vdots \end{array} \right.$	12,14 $\sim E$
16	(16)	$\left[\begin{array}{l} \sim C \\ \vdots \end{array} \right.$	<i>Antagande</i>
1, 3, 16	(17)	$\left[\begin{array}{l} \lambda \\ \vdots \end{array} \right.$	11,16 $\sim E$
1, 2, 3, 4	(18)	$\left[\begin{array}{l} \lambda \\ \vdots \end{array} \right.$	2,12,15,16,17 $\vee E$
1, 2, 3	(19)	$\left[\begin{array}{l} \sim(A \leftrightarrow B) \\ \vdots \end{array} \right.$	4,18 $\sim I$
1, 2	(20)	$A \rightarrow \sim(A \leftrightarrow B)$	3,19 $\rightarrow I$

4.8.6 Visa $\sim(A \rightarrow B \vee C) \vdash B \vee C \rightarrow A$

1	(1)	$\sim(A \rightarrow B \vee C)$	<i>Premiss</i>
1	(2)	$A \& \sim(B \vee C)$	1 <i>Neg - Imp</i>
1	(3)	A	2 $\& E$
4	(4)	$\left[\begin{array}{l} B \vee C \\ \vdots \end{array} \right.$	<i>Antagande</i>
1	(5)	$\left[\begin{array}{l} A \\ \vdots \end{array} \right.$	3
1	(6)	$B \vee C \rightarrow A$	$\rightarrow 4, 5$

4.11.2 a. Mängden är inte konsistent. Det visa på följande sätt:

1	(1)	$A \vee B$	<i>Premiss</i>
2	(2)	$\sim A \rightarrow \sim B$	<i>Premiss</i>
3	(3)	$\sim A$	<i>Premiss</i>
2, 3	(4)	$\sim B$	2,3 $\rightarrow E$
5	(5)	$\left[\begin{array}{l} A \\ \vdots \end{array} \right.$	<i>Antagande</i>
3, 5	(6)	$\left[\begin{array}{l} \lambda \\ \vdots \end{array} \right.$	3,5 $\sim E$
7	(7)	$\left[\begin{array}{l} B \\ \vdots \end{array} \right.$	<i>Antagande</i>
2, 3, 7	(8)	$\left[\begin{array}{l} \lambda \\ \vdots \end{array} \right.$	4,7 $\sim E$
1, 2, 3	(9)	λ	1,5,6,7,8 $\vee E$

b. Mängden är konsistent. Om A, B, C alla är falska blir alla fyra sentenserna sanna.

c. Mängden är inte konsistent. Det visa på följande sätt:

1	(1)	$A \vee B$	<i>Premiss</i>
2	(2)	$\sim A \vee \sim B$	<i>Premiss</i>
3	(3)	$A \leftrightarrow B$	<i>Premiss</i>
3	(4)	$(A \rightarrow B) \& (B \rightarrow A)$	3 Df
5	(5)	A	<i>Antagande</i>
3	(6)	$A \rightarrow B$	4 &E
3, 5	(7)	B	5,6 \rightarrow E
8	(8)	$\sim A$	<i>Antagande</i>
5, 8	(9)	\wedge	5,8 \sim E
10	(10)	$\sim B$	<i>Antagande</i>
3, 5, 10	(11)	\wedge	7,10 \sim E
2, 3, 5	(12)	\wedge	2,8,9,10,11 \vee E
2, 3	(13)	$\sim A$	5,12 \sim I
14	(14)	A	<i>Antagande</i>
2, 3, 14	(15)	\wedge	13,14 \sim E
16	(16)	B	<i>Antagande</i>
3	(17)	$B \rightarrow A$	4 &E
3, 16	(18)	A	16,17 \rightarrow E
2, 3, 16	(19)	\wedge	13,18 \sim E
1, 2, 3	(20)	\wedge	1,14,15,16,19 \vee E