Cox Risk Processes and Ruin

Hanspeter Schmidli

University of Cologne

Risk Modelling in Insurance and Finance in Honour of Jan Grandell's Birthday Stockholm, 13th of June

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Generalisations

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The Classical Theory

- The Cramér–Lundberg Model
- The Sparre–Andersen Model



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 - The Sparre–Andersen Model
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 - The Björk–Grandell Model
 - The Markov-Modulated Model
 - Cox Models

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 - Cox Models

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 - Heavy Tails

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The Cramér–Lundberg Model

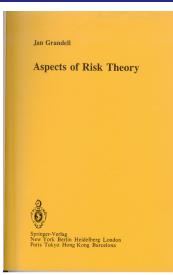
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The Cramér–Lundberg Model			

Assumptions

We assume in this talk that all risk processes converge to ∞ and that all quantities defined are well-defined.

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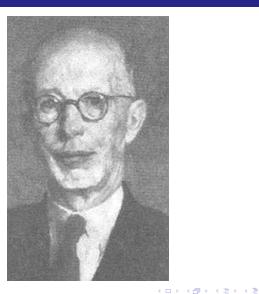
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The Cramér–Lundberg Model			

$$X_t = \mathbf{x} + ct - \sum_{i=1}^{N_t} Y_i$$

• x: initial capital

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The Cramér–Lundberg Model			

$$X_t = x + ct - \sum_{i=1}^{N_t} Y_i$$

- x: initial capital
- c: premium rate

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$$X_t = x + ct - \sum_{i=1}^{N_t} Y_i$$

- x: initial capital
- c: premium rate
- $\{N_t\}$: Poisson process with rate λ

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$$X_t = x + ct - \sum_{i=1}^{N_t} \frac{Y_i}{Y_i}$$

- x: initial capital
- c: premium rate
- $\{N_t\}$: Poisson process with rate λ
- {**Y**_i}: iid,

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- x: initial capital
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- $\{N_t\}$: Poisson process with rate λ
- $\{Y_i\}$: iid, independent of $\{N_t\}$
- G(y): distribution function of Y_i , G(0) = 0

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The Cramér–Lundberg Model			

$$X_t = x + ct - \sum_{i=1}^{N_t} \frac{\mathbf{Y}_i}{\mathbf{Y}_i}$$

- x: initial capital
- c: premium rate
- $\{N_t\}$: Poisson process with rate λ
- $\{Y_i\}$: iid, independent of $\{N_t\}$
- G(y): distribution function of Y_i , G(0) = 0
- $\mu_n = \mathbb{E}[Y_i^n], \quad \mu = \mu_1, \quad M_Y(r) = \mathbb{E}[e^{rY}].$

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Let

 $\theta(r) = \lambda(M_Y(r) - 1) - cr .$



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The Cramér–Lundberg Model			

Let

$$\theta(r) = \lambda(M_Y(r) - 1) - cr$$
.

Then $\{e^{-r(X_t-x)-\theta(r)t}\}$ is a martingale with mean value 1.

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The Cramér–Lundberg Model			

Let

$\theta(r) = \lambda(M_Y(r) - 1) - cr$.

Then $\{e^{-r(X_t-x)-\theta(r)t}\}\$ is a martingale with mean value 1. Define the measure Q_r as

$$Q_r[A] = \mathbb{E}[\mathrm{e}^{-r(X_T - x) - \theta(r)T}; A], \qquad A \in \mathcal{F}_T \cap \{T < \infty\}$$

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Here T is a stopping time.

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Let

$\theta(r) = \lambda(M_Y(r) - 1) - cr$.

Then $\{e^{-r(X_t-x)-\theta(r)t}\}\$ is a martingale with mean value 1. Define the measure Q_r as

 $Q_r[A] = \operatorname{I\!E}[\mathrm{e}^{-r(X_T - x) - \theta(r)T}; A] , \qquad A \in \mathcal{F}_T \cap \{T < \infty\} .$

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Here T is a stopping time. We let R be the strict positive solution to $\theta(r) = 0$.

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Under Q_r the model remains a Cramér–Lundberg model with intensity $\lambda_r = \lambda M_Y(r)$ and claim size distribution

$$Q_r[Y \leq x] = rac{\lambda}{ heta(r) + cr + \lambda} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \; .$$

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For the net profit we get $\mathbb{E}_r[X_t^1 - x] = -\theta'(r)$.

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For the net profit we get $\mathbb{E}_r[X_t^1 - x] = -\theta'(r)$. In particular, $Q_r[\liminf_{t\to\infty} X_t = -\infty] = 1$ for $\theta'(r) \ge 0$.

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Ruin Probabilities

Let $\tau = \inf\{t : X_t < 0\}$. We let

 $\psi(x;t) = \operatorname{I\!P}[\tau \leq t] \; .$



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Then we find

$$\psi(x; t) = \mathbb{P}[\tau \le t] = \mathbb{E}_r[e^{rX_\tau + \theta(r)\tau}; \tau \le t]e^{-rx}$$

and

$$\psi(x) = \mathbb{P}[\tau < \infty] = \mathbb{E}_{R}[\mathrm{e}^{RX_{\tau}}; \tau < \infty]\mathrm{e}^{-Rx}$$

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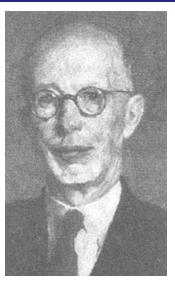
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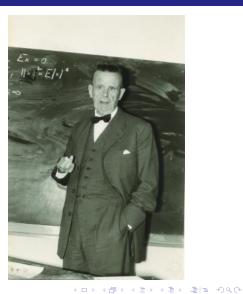
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Lundberg Inequalities

From $X_{ au} < 0$ we get

$$\psi(\mathbf{x}) = \mathbb{E}_{R}[\mathrm{e}^{RX_{\tau}}]\mathrm{e}^{-R\mathbf{x}} < \mathrm{e}^{-R\mathbf{x}} ,$$

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$$\psi(x;\underline{y}x) = \mathbb{E}_r[\mathrm{e}^{rX_\tau + \theta(r)\tau}; \tau \leq \underline{y}x]\mathrm{e}^{-rx} < \mathrm{e}^{-\min\{r - \theta(r)\underline{y}, r\}x}.$$

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Let $\underline{R} = \sup\{r - \theta(r)\underline{y}:r \geq R\}$. Then

 $\psi(x;\underline{y}x) < \mathrm{e}^{-\underline{R}x}$.

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Let $\underline{R} = \sup\{r - \theta(r)\underline{y} : r \ge R\}$. Then

$$\psi(x; \underline{y}x) < \mathrm{e}^{-\underline{R}x}$$
 .

Note that $\underline{R} \geq R$.

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Lundberg Inequalities

Analogously,

$$\psi(x) - \psi(x; \bar{y}x) < e^{-\bar{R}x}$$

for $\overline{R} = \sup\{r - \theta(r)\overline{y} : r \leq R\}.$



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Analogously,

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for $\overline{R} = \sup\{r - \theta(r)\overline{y} : r \leq R\}$. Also here, $\overline{R} \geq R$.

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for $\overline{R} = \sup\{r - \theta(r)\overline{y} : r \leq R\}$. Also here, $\overline{R} \geq R$. It turns out that $\underline{R} > R$ if $\underline{y} < y_0$ and $\overline{R} > R$ if $\overline{y} > y_0$ for the critical value

$$y_0 = \frac{1}{\theta'(R)} = \frac{1}{\lambda M'_{\gamma}(R) - c}$$

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The Cramér–Lundberg Model

Lundberg Inequalities

Analogously,

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for $\overline{R} = \sup\{r - \theta(r)\overline{y} : r \leq R\}$. Also here, $\overline{R} \geq R$. It turns out that $\underline{R} > R$ if $\underline{y} < y_0$ and $\overline{R} > R$ if $\overline{y} > y_0$ for the critical value

$$y_0 = rac{1}{ heta'(R)} = rac{1}{\lambda M'_Y(R) - c} \, .$$

Moreover,

$$\frac{\tau}{x} \xrightarrow{\mathrm{P}} y_0$$

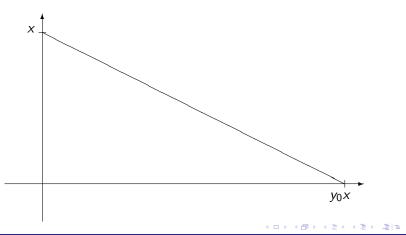
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Path given $\{\tau < \infty\}$



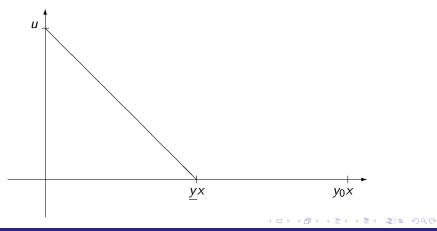
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The Cramér–Lundberg Model

Path given $\{\tau \leq \underline{y}x\}$



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The Cramér–Lundberg Model

The Cramér–Lundberg Approximation

From the above considerations we see that

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\lim_{x\to\infty}\operatorname{I\!E}_R[\mathrm{e}^{RX_\tau}]\,.$$

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The Cramér–Lundberg Model

The Cramér–Lundberg Approximation

From the above considerations we see that

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\lim_{x\to\infty}\operatorname{I\!E}_R[\mathrm{e}^{RX_\tau}].$$

By considering the ladder times, the function $f(x) = \mathbb{E}_R[e^{RX_\tau} \mid X_0 = x]$ fulfils a renewal equation. By the key renewal theorem we get

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\frac{c-\lambda\mu}{\lambda M'_Y(R)-c}\;.$$

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The Sparre-Andersen Model

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The Sparre-Andersen Model

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The Sparre-Andersen Model

The Sparre–Andersen Risk Model

$$X_t = x + ct - \sum_{i=1}^{N_t} Y_i$$

- x: initial capital
- c: premium rate
- $\{N_t\}$: Ordinary renewal process
- $\{Y_i\}$: iid, independent of $\{N_t\}$

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The Sparre-Andersen Model

The Sparre–Andersen Risk Model

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- F(y): distribution function of $T_i T_{i-1}$

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Generalisations

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The Sparre-Andersen Model

The Sparre–Andersen Risk Model

$$X_t = x + ct - \sum_{i=1}^{N_t} Y_i$$

- x: initial capital
- c: premium rate
- $\{N_t\}$: Ordinary renewal process
- $\{Y_i\}$: iid, independent of $\{N_t\}$
- F(y): distribution function of $T_i T_{i-1}$
- $\lambda = (\mathbb{E}[T_i T_{i-1}])^{-1}, \qquad M_T(r) = \mathbb{E}[\mathrm{e}^{r(T_i T_{i-1})}].$

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The Sparre–Andersen Model			
Markovisation			

Let $A_t = T_{N_t+1} - t$ be the time to the next claim. Then $\{X_t, A_t\}$ is a Markov process.



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Markovisation

Let $A_t = T_{N_t+1} - t$ be the time to the next claim. Then $\{X_t, A_t\}$ is a Markov process. Let $\theta(r)$ be the unique solution to $M_Y(r)M_T(-\theta - cr) = 1$. Then the process

 $\{M_Y(r)\mathrm{e}^{-r(X_t-x)-(\theta(r)+cr)A_t-\theta(r)t}\}$

is a martingale.

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Markovisation

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is a martingale. Define the measure Q_r as

$$Q_r[A] = \operatorname{I\!E}[M_Y(r) \mathrm{e}^{-r(X_T - x) - (\theta(r) + cr)A_T - \theta(r)T}; A] \; .$$

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Markovisation

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 $Q_r[A] = \mathbb{E}[M_Y(r) \mathrm{e}^{-r(X_T - x) - (\theta(r) + cr)A_T - \theta(r)T}; A] \; .$

We denote by *R* the positive solution to $\theta(r) = 0$. I.e., the solution to $M_Y(r)M_T(-cr) = 1$.

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The Sparre-Andersen Model			

Under Q_r the process remains a Sparre-Andersen model with

$$M_Y(r) = M_T(-\theta(r) - cr) \int_0^x \mathrm{e}^{ry} \mathrm{d}G(y) \; ,$$

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$$M_T(r) = M_Y(r) \int_0^t \mathrm{e}^{-(heta(r)+cr)s} \,\mathrm{d}F(s) \;.$$

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Ruin occurs almost surely if $\theta'(r) \ge 0$.

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The Sparre–Andersen Model			

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Ruin occurs almost surely if $\theta'(r) \ge 0$. The ruin probabilities can be expressed as

 $\psi(x) = M_T(-cR) \mathbb{I}_R[e^{RX_\tau + cRA_\tau}]e^{-Rx}$

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The Sparre–Andersen Model			

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Ruin occurs almost surely if $\theta'(r) \ge 0$. The ruin probabilities can be expressed as

$$\psi(\mathbf{x}) = M_T(-cR)\mathbb{E}_R[\mathrm{e}^{RX_\tau + cRA_\tau}]\mathrm{e}^{-R\mathbf{x}} = \mathbb{E}_R[\mathrm{e}^{RX_\tau}]\mathrm{e}^{-R\mathbf{x}},$$

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The Sparre–Andersen Model			

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$$\psi(\mathsf{x};t) = \mathbb{E}_r[\mathrm{e}^{rX_ au+ heta(r) au}; au \leq t]\mathrm{e}^{-r\mathsf{x}}$$
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The Classical Theory			Minimal Ruin Probabilities
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Lundberg Inequalities

As in the classical model we find

 $\psi(x) < \mathrm{e}^{-Rx} \; ,$

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The Classical Theory			Minimal Ruin Probabilities
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Lundberg Inequalities

As in the classical model we find

 $\psi(x) < \mathrm{e}^{-Rx} \; ,$

 $\psi(x;\underline{y}x) < \mathbb{E}_r[\mathrm{e}^{\theta(r)\tau}; \tau \leq \underline{y}x]\mathrm{e}^{-rx} < \mathrm{e}^{-\min\{r-\theta(r)\underline{y},r\}x}$.

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The Classical Theory			Minimal Ruin Probabilities
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Lundberg Inequalities

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$$\begin{split} \psi(x;\underline{y}x) < \mathbb{E}_r[\mathrm{e}^{\theta(r)\tau};\tau \leq \underline{y}x]\mathrm{e}^{-rx} < \mathrm{e}^{-\min\{r-\theta(r)\underline{y},r\}x} \ . \end{split}$$
 Again choose $\underline{R} = \sup\{r - \theta(r)\underline{y}:r \geq R\}$. Then $\psi(x;yx) < \mathrm{e}^{-\underline{R}x} \ . \end{split}$

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 Again choose $\underline{R} = \sup\{r - \theta(r)\underline{y}:r \geq R\}$. Then $\psi(x;\underline{y}x) < \mathrm{e}^{-\underline{R}x} \ . \end{split}$

Note that $\underline{R} \geq R$.

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The Classical Theory			Minimal Ruin Probabilities
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 $\psi(x) < \mathrm{e}^{-Rx} ,$

$$\begin{split} \psi(x;\underline{y}x) < \mathbb{E}_r[\mathrm{e}^{\theta(r)\tau};\tau \leq \underline{y}x]\mathrm{e}^{-rx} < \mathrm{e}^{-\min\{r-\theta(r)\underline{y},r\}x} \ . \end{split}$$
Again choose $\underline{R} = \sup\{r - \theta(r)\underline{y}:r \geq R\}$. Then $\psi(x;yx) < \mathrm{e}^{-\underline{R}x} \ . \end{split}$

Note that $\underline{R} \geq R$. Analogously

$$\psi(x) - \psi(x; \bar{y}x) < \mathrm{e}^{-\bar{R}x}$$
.

for $\overline{R} = \sup\{r - \theta(r)\overline{y} : r \leq R\} \geq R$.

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Cox Risk Processes and Ruin

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The Classical Theory			Minimal Ruin Probabilities
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Lundberg Inequalities

It again turns out that $\underline{R} > R$ if $\underline{y} < y_0$ and $\overline{R} > R$ if $\overline{y} > y_0$ for the critical value

$$y_0 = \frac{1}{\theta'(R)} = \left(\frac{M'_Y(R)M_T(-cR)}{M_Y(R)M'_T(-cR)} - c\right)^{-1}.$$

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The Classical Theory			Minimal Ruin Probabilities
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Lundberg Inequalities

It again turns out that $\underline{R} > R$ if $\underline{y} < y_0$ and $\overline{R} > R$ if $\overline{y} > y_0$ for the critical value

$$y_0 = \frac{1}{\theta'(R)} = \left(\frac{M'_Y(R)M_T(-cR)}{M_Y(R)M'_T(-cR)} - c\right)^{-1}.$$

Moreover,

$$\frac{\tau}{x} \xrightarrow{\mathrm{P}} y_0$$

on $\{\tau < \infty\}$.

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The Cramér–Lundberg Approximation

From the above considerations we see that

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\lim_{x\to\infty}\operatorname{I\!E}_R[\mathrm{e}^{RX_\tau}]\,.$$

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Generalisations

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The Sparre–Andersen Model

The Cramér–Lundberg Approximation

From the above considerations we see that

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\lim_{x\to\infty}\operatorname{I\!E}_R[\mathrm{e}^{RX_\tau}]\,.$$

It follows again by a renewal approach that

$$\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=C\;,$$

where $C \ge 0$ is some constant. If $\theta'(R) < \infty$ then C > 0.

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Generalisations

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The Björk-Grandell Model

The Classical Theory
The Cramér–Lundberg Model
The Sparre–Andersen Model

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 - The Björk–Grandell Model
 - The Markov-Modulated Model
 - Cox Models
- 3 Heavy Tails
 - The Classical Models
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 - Cramér–Lundberg Approximations
 - Heavy Tails

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Generalisations

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The Björk-Grandell Model

The Ammeter Risk Model

Let $\{L_i\}$ be a sequence of iid positive random variables. On the interval [i - 1, i) let $\{X_t\}$ behave like a classical risk model with claim intensity L_i and claim size distribution G(y).

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Hanspeter Schmidli Cox Risk Processes and Ruin

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The Björk-Grandell Model

The Ammeter Risk Model

Let $\{L_i\}$ be a sequence of iid positive random variables. On the interval [i - 1, i) let $\{X_t\}$ behave like a classical risk model with claim intensity L_i and claim size distribution G(y).

If L_i has a Gamma distribution then N_1 is negative binomially distributed.

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The Biörk–Grandell Model

Generalisations

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The Björk–Grandell Model

Let $\{L_i, \sigma_i\}$ be a sequence of iid random vectors with distribution function $F(\ell, s)$, where $L_i \ge 0$ and $\sigma_i > 0$. We denote by $S_i = \sum_{k=1}^{i} \sigma_k$. On the interval $[S_{i-1}, S_i)$ let $\{X_t\}$ behave like a classical risk model with claim intensity L_i and claim size distribution G(y).

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Generalisations

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The Björk–Grandell Model

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We let $\lambda_t = L_i$ and $A_t = S_i - t$ if $S_{i-1} \le t \le S_i$. Then $\{(X_t, \lambda_t, A_t) \text{ is a Markov process.}\}$

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The Classical Theory	Generalisations		Minimal Ruin Probabilities
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The Biörk–Grandell Model			

Change of Measure

Consider the function

$$\varphi(\vartheta, r) := \mathbb{E}[\exp\{[L(M_Y(r) - 1) - \vartheta - cr -]\sigma\}].$$

We let $\theta(r)$ be the unique solution to $\varphi(\theta(r), r) = 1$ and R be the strictly positive solution to $\theta(r) = 0$.

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The Biörk–Grandell Model			

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We let $\theta(r)$ be the unique solution to $\varphi(\theta(r), r) = 1$ and R be the strictly positive solution to $\theta(r) = 0$. The process

$$\left\{ e^{-r(X_t-x)+(\lambda_t(M_Y(r)-1)-cr-\theta(r))A_t-\theta(r)t} \right\}$$

is a martingale.

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The Classical Theory	Generalisations		Minimal Ruin Probabilities
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The Björk–Grandell Model			

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We let $\theta(r)$ be the unique solution to $\varphi(\theta(r), r) = 1$ and R be the strictly positive solution to $\theta(r) = 0$. The process

$$\left\{ e^{-r(X_t-x)+(\lambda_t(M_Y(r)-1)-cr-\theta(r))A_t-\theta(r)t} \right\}$$

is a martingale. We define the measure

$$Q_r[A] = \operatorname{I\!E}[\mathrm{e}^{-r(X_t - x) + (\lambda_t(M_Y(r) - 1) - cr - \theta(r))A_t - \theta(r)t}; A] \; .$$

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The Classical Theory 00000000000000 0000000	Generalisations 0000000000 0000000000000000000000000	Heavy Tails 00 0000	Minimal Ruin Probabilities 00000 000000000 00000000
The Biörk–Grandell Model	0000000000		0000000

Under the new measure the process is again a Björk–Grandell model with

$$Q_r[Y \leq x] = \frac{1}{M_Y(r)} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \; ,$$

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Cox Risk Processes and Ruin

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The Biörk-Grandell Model			

Under the new measure the process is again a Björk–Grandell model with

$$Q_r[Y \leq x] = \frac{1}{M_Y(r)} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \; ,$$

$$Q_r[L \leq \ell, \sigma \leq s] = \int_0^\ell \int_0^s e^{[I(M_Y(r)-1)-cr-\theta(r)]w} F(\mathrm{d}I, \mathrm{d}w)$$

The claim intensity is $\lambda_t M_Y(r)$.

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The Biörk-Grandell Model			

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$$Q_r[Y \leq x] = \frac{1}{M_Y(r)} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \; ,$$

$$Q_r[L \leq \ell, \sigma \leq s] = \int_0^\ell \int_0^s e^{[I(M_Y(r)-1)-cr-\theta(r)]w} F(\mathrm{d}I, \mathrm{d}w)$$

The claim intensity is $\lambda_t M_Y(r)$. The drift is $-\theta'(r)$.

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The Björk–Grandell Model			

The Ruin Probabilities

We can express the ruin probabilities as

$$\psi(x) = \mathbb{E}_{R}[\mathrm{e}^{RX_{\tau} - (\lambda_{\tau}(M_{Y}(R) - 1) - cR)A_{\tau}}]\mathrm{e}^{-Rx}.$$

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The Biörk–Grandell Model			

The Ruin Probabilities

We can express the ruin probabilities as

$$\psi(x) = \operatorname{I\!E}_R[\mathrm{e}^{RX_\tau - (\lambda_\tau (M_Y(R) - 1) - cR)A_\tau}] \mathrm{e}^{-Rx} \ .$$

The finite time ruin probability becomes

$$\psi(\mathsf{x};t) = \mathbb{E}_r[\mathrm{e}^{r(X_\tau-\mathsf{x})-\{\lambda_\tau(M_Y(r)-1)-cr-\theta(r)\}A_\tau+\theta(r)\tau};\tau\leq t]\mathrm{e}^{-r\mathsf{x}}.$$

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The Björk–Grandell Model			



We need the following assumption: Suppose there is a constant B > 0 such that

 $\mathbb{E}_{\mathbb{P}}[\mathrm{e}^{\{L(M_{Y}(r)-1)-cr-\theta(r)\}(\sigma-\nu)} \mid \sigma > \nu, L] \ge B \quad (a.s.). \qquad (\mathscr{A}_{r})$ for all $\nu \ge 0$.

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The Björk–Grandell Model			



We need the following assumption: Suppose there is a constant B > 0 such that

 $\mathbb{E}_{\mathbb{P}}[e^{\{L(M_Y(r)-1)-cr-\theta(r)\}(\sigma-\nu)} \mid \sigma > \nu, L] \ge B \quad (a.s.). \tag{Ar}$

for all $v \ge 0$.

The condition is fulfilled if $\mathbb{E}_{\mathrm{IP}}[\sigma - v \mid \sigma > v, L = \ell] < \infty$ for all $\ell \leq (M_Y(r) - 1)^{-1}(cr + \theta(r)).$

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The Biörk-Grandell Model			

Lundberg Inequalities

Under the assumption (\mathscr{A}_R) one has

 $\limsup_{x\to\infty}\psi(x)\mathrm{e}^{Rx}<\infty\;.$



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Lundberg Inequalities

Under the assumption (\mathscr{A}_R) one has

 $\limsup_{x\to\infty}\psi(x)\mathrm{e}^{Rx}<\infty\;.$

Under the assumption $(\mathscr{A}_{\underline{R}})$ one has

 $\limsup_{x\to\infty}\psi(x;\underline{y}x)\mathrm{e}^{\underline{R}x}<\infty\;.$

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The Biörk-Grandell Model			

Lundberg Inequalities

Under the assumption (\mathscr{A}_R) one has

 $\limsup_{x\to\infty}\psi(x)\mathrm{e}^{Rx}<\infty\;.$

Under the assumption $(\mathscr{A}_{\underline{R}})$ one has

 $\limsup_{x\to\infty}\psi(x;\underline{y}x)\mathrm{e}^{\underline{R}x}<\infty.$

Under the assumption $(\mathscr{A}_{\overline{R}})$ one has

$$\limsup_{x\to\infty}(\psi(x)-\psi(x;\bar{y}x))e^{\bar{R}x}<\infty.$$

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Here \underline{R} and \overline{R} are defined as above.

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The Björk-Grandell Model

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Cramér–Lundberg Approximation

Also here we get under $(\mathscr{A}_{\underline{R}})$ and $(\mathscr{A}_{\overline{R}})$ the limit

 $\frac{\tau}{x} \xrightarrow{\mathrm{P}} y_0$

on $\{\tau < \infty\}$.

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The Björk–Grandell Model

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Cramér–Lundberg Approximation

Also here we get under $(\mathscr{A}_{\underline{R}})$ and $(\mathscr{A}_{\overline{R}})$ the limit

 $\frac{\tau}{x} \xrightarrow{\mathrm{P}} y_0$

on $\{\tau < \infty\}$.

If (\mathscr{A}_r) holds for some r > R then there is a constant C > 0 such that

 $\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=C\;.$

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The Markov-Modulated Model

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The Cramér–Lundberg Model
The Sparre–Andersen Model

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The Markov-Modulated Model

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Generalisations

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The Markov-Modulated Model

The Markov-Modulated Model

Let $\{J_t\}$ be a Markov chain with state space $\{1, 2, \ldots, \mathscr{J}\}$ and intensity matrix $\eta = (\eta_{ij})$. On $\{J_t = i\}$ the process $\{X_t\}$ behaves like a classical model with intensity L_i and claim size distribution G_i . We denote by $\{\pi_i\}$ the stationary distribution of $\{J_t\}$.

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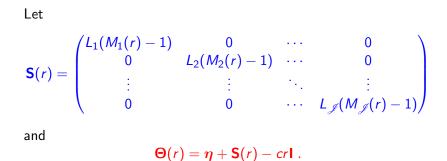
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The Markov-Modulated Model

The Markov-Modulated Model



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The Classical Theory 00000000000000 00000000	Generalisations 000000000 0000000 000000000000000000	Heavy Tails 00 0000	Minimal Ruin Probabilities 00000 000000000 00000000
The Markov-Modulated Model			

The Martingale

We have

$$\mathbb{E}[e^{-r(X_t-x)}\mathbb{1}_{\{J_t=j\}} \mid J_0=i] = (e^{t\Theta(r)})_{ij} .$$

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The Classical Theory 00000000000000 00000000	Generalisations 000000000 00000000000000000000000000	Heavy Tails 00 0000	Minimal Ruin Probabilities 00000 000000000 00000000
The Markov-Modulated Model			

The Martingale

We have

$$\operatorname{I\!E}[\mathrm{e}^{-r(X_t-x)} 1\!\!1_{\{J_t=j\}} \mid J_0=i] = (\mathrm{e}^{t\Theta(r)})_{ij} \; .$$

Let $\theta(r)$ be the eigenvalue of $\Theta(r)$ with the largest real part and $\mathbf{g}(r)$ be the corresponding eigenvector $(g_i(r) > 0)$. Then

$$\frac{g_{J_t}(r)}{\mathbb{E}[g_{J_0}(r)]} e^{-r(X_t-x)-\theta(r)t}$$

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Generalisations

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The Markov-Modulated Model

The Change of Measure

Define as before

$$Q_r[A] = \mathbb{E}\Big[\frac{g_{J_t}(r)}{\mathbb{E}[g_{J_0}(r)]} e^{-r(X_t - x) - \theta(r)t}; A\Big] .$$

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The Classical Theory	Generalisations		Minimal Ruin Probabilities
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The Markov-Modulated Model

The Change of Measure

Define as before

$$Q_r[A] = \mathbb{E}\Big[\frac{g_{J_t}(r)}{\mathbb{E}[g_{J_0}(r)]} e^{-r(X_t - x) - \theta(r)t}; A\Big] .$$

Under Q_r the process $\{(X_t, J_t)\}$ remains a Markov modulated risk model with claim intesities $L_i M_i(r)$, claim size distribution

$$Q_t[Y \leq y \mid J=i] = (M_i(r))^{-1} \int_0^y \mathrm{e}^{rz} \, \mathrm{d}G_i(z)$$

and intensity matrix $diag((g_i(r))^{-1})\eta diag(g_i(r))$.

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Cox Risk Processes and Ruin

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The Markov-Modulated Model

The Change of Measure

Define as before

$$Q_r[A] = \mathbb{E}\Big[\frac{g_{J_t}(r)}{\mathbb{E}[g_{J_0}(r)]} e^{-r(X_t - x) - \theta(r)t}; A\Big] .$$

Under Q_r the process $\{(X_t, J_t)\}$ remains a Markov modulated risk model with claim intesities $L_i M_i(r)$, claim size distribution

$$Q_t[Y \leq y \mid J = i] = (M_i(r))^{-1} \int_0^y e^{rz} \, \mathrm{d}G_i(z)$$

and intensity matrix $\operatorname{diag}((g_i(r))^{-1})\eta \operatorname{diag}(g_i(r))$. In particular, the drift is $-\theta'(r)$.

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The Ruin Probabilities

Let *R* be the strictly positive solution to $\theta(r) = 0$. Then

$$\psi(x) = \mathbb{E}_{\mathbb{P}}[g_{J_0}(R)]\mathbb{E}_R\left[\frac{\mathrm{e}^{RX_{\tau}}}{g_{J_{\tau}(R)}}\right]\mathrm{e}^{-Rx} ,$$

$$\psi(x;t) = \mathbb{E}_{\mathrm{I\!P}}[g_{J_0}(r)]\mathbb{E}_r\Big[rac{\mathrm{e}^{rX_ au+ heta(r) au}}{g_{J_ au(r)}}; au\leq t\Big]\mathrm{e}^{-rx}\;.$$

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The Classical Theory	Generalisations		Minimal Ruin Probabilities
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The Ruin Probabilities

Let *R* be the strictly positive solution to $\theta(r) = 0$. Then

$$\psi(x) = \mathbb{E}_{\mathbb{P}}[g_{J_0}(R)] \mathbb{E}_R \Big[\frac{\mathrm{e}^{R X_{\tau}}}{g_{J_{\tau}(R)}} \Big] \mathrm{e}^{-R x} ,$$

$$\psi(x;t) = \mathbb{E}_{\mathbb{IP}}[g_{J_0}(r)]\mathbb{E}_r\Big[rac{\mathrm{e}^{rX_ au+ heta(r) au}}{g_{J_ au(r)}}; au\leq t\Big]\mathrm{e}^{-rx}\;.$$

Lundberg inequalities and Cramér–Lundberg approximation follow as before.

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Generalisations

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Cox Models

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Cox Models

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Cox Model with Piecewise Constant Intensities

Let $\{(J_i, \sigma_i)\}$ be some Markov chain (with infinite state space), where $\sigma_i > 0$. We define $S_i = \sum_{j=1}^{i} \sigma_i$. On $[S_{i-1}, S_i)$ the process $\{X_t\}$ behaves like a classical model with intensity $L(J_i) \ge 0$ and claim size distribution $G(y; J_i)$.

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Cox Models

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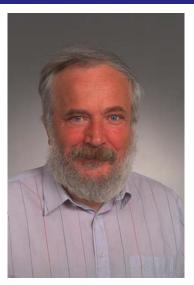
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Cox Model with Piecewise Constant Intensities

Let $\{(J_i, \sigma_i)\}$ be some Markov chain (with infinite state space), where $\sigma_i > 0$. We define $S_i = \sum_{j=1}^{i} \sigma_i$. On $[S_{i-1}, S_i)$ the process $\{X_t\}$ behaves like a classical model with intensity $L(J_i) \ge 0$ and claim size distribution $G(y; J_i)$.

Under additional conditions (for ergodicity) the Lundberg inequalities and the Cramér–Lundberg approximation holds.

Generalisations



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Diffusion Intensities

Let $\{Z_t\}$ be a diffusion process following the stochastic differential equation

$\mathrm{d} Z_t = b(Z_t) \, \mathrm{d} W_t + a(Z_t) \, \mathrm{d} t$

for some Brownian motion $\{W_t\}$. The claim number process $\{N_t\}$ is a compound Poisson process with rate $\{\ell(Z_t)\}$.

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Diffusion Intensities

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$\mathrm{d}Z_t = b(Z_t) \,\mathrm{d}W_t + a(Z_t) \,\mathrm{d}t$

for some Brownian motion $\{W_t\}$. The claim number process $\{N_t\}$ is a compound Poisson process with rate $\{\ell(Z_t)\}$.

The process $\{g(Z_t)e^{-r(X_t-x)-\theta(r)t}\}$ is a martingale if

 $\frac{1}{2}b^2(z)g''(z) + a(z)g'(z) + [\ell(z)(M_Y(r) - 1) - \theta - cr -]g(z) = 0.$

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Generalisations

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Cox Models

The Change of Measure

Consider the measure

$$Q_r[A] = \frac{\mathbb{E}[g(Z_T)e^{-r(X_T-x)-\theta(r)T}; A]}{\mathbb{E}[g(Z_0)]} .$$



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The Change of Measure

Consider the measure

$$Q_r[A] = rac{\operatorname{I\!E}[g(Z_T) \mathrm{e}^{-r(X_T - x) - heta(r)T}; A]}{\operatorname{I\!E}[g(Z_0)]} \; .$$

The process $({X_t, Z_t})$ remains a Cox model with claim size distribution

$$Q[Y \leq x] = (M_Y(r))^{-1} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \ ,$$

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The Change of Measure

Consider the measure

$$Q_r[A] = \frac{\mathbb{E}[g(Z_T)e^{-r(X_T-x)-\theta(r)T}; A]}{\mathbb{E}[g(Z_0)]} .$$

The process $({X_t, Z_t})$ remains a Cox model with claim size distribution

$$Q[Y \leq x] = (M_Y(r))^{-1} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \,,$$

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claim intensity $\ell(Z_t)M_Y(r)$,

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The Change of Measure

Consider the measure

$$Q_r[A] = \frac{\mathbb{E}[g(Z_T)e^{-r(X_T-x)-\theta(r)T}; A]}{\mathbb{E}[g(Z_0)]} .$$

The process $({X_t, Z_t})$ remains a Cox model with claim size distribution

$$Q[Y \leq x] = (M_Y(r))^{-1} \int_0^x \mathrm{e}^{ry} \, \mathrm{d}G(y) \; ,$$

claim intensity $\ell(Z_t)M_Y(r)$, and generator of the diffusion

$$\tilde{\mathfrak{A}}f = \frac{ga + b^2g'}{g}f' + \frac{1}{2}b^2f'' \; .$$

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Cox Models			

The Ruin Probabilities

The drift of $\{X_t\}$ is again $-\theta'(r)$.



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The Ruin Probabilities

The drift of $\{X_t\}$ is again $-\theta'(r)$. Suppose there is a R > 0 such that $\theta(R) = 0$. The ruin probabilities can be expressed as

$$\psi(x) = \mathbb{E}_{\mathbb{IP}}[g(Z_0)]\mathbb{E}_R\Big[rac{\mathrm{e}^{RX_{\tau}}}{g(Z_{\tau})}\Big]\mathrm{e}^{-Rx} \; ,$$

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The Ruin Probabilities

The drift of $\{X_t\}$ is again $-\theta'(r)$. Suppose there is a R > 0 such that $\theta(R) = 0$. The ruin probabilities can be expressed as

$$\psi(x) = \mathbb{E}_{\mathbb{P}}[g(Z_0)]\mathbb{E}_R\left[\frac{\mathrm{e}^{RX_{\tau}}}{g(Z_{\tau})}\right]\mathrm{e}^{-Rx} ,$$

$$\psi(x;t) = \mathbb{E}_{\mathbb{IP}}[g(Z_0)]\mathbb{E}_R\Big[rac{\mathrm{e}^{rX_ au+ heta(r) au}}{g(Z_ au)}; au \leq t\Big]\mathrm{e}^{-rx} \; .$$

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If g(z) is bounded away from zero we obtain as before

 $\psi(x) < \frac{\operatorname{I\!E}_{\operatorname{I\!P}}[g(Z_0)]}{\inf g(z)} \mathrm{e}^{-Rx} \; ,$



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Cox Models			

If g(z) is bounded away from zero we obtain as before

$$\psi(x) < \frac{\operatorname{I\!E}_{\operatorname{I\!P}}[g(Z_0)]}{\inf g(z)} e^{-Rx} ,$$

$$\psi(x; \underline{y}x) < \frac{\operatorname{IE}_{\operatorname{IP}}[g(Z_0)]}{\inf g(z)} \mathrm{e}^{-\underline{R}x}$$

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Cox Models			

If g(z) is bounded away from zero we obtain as before

$$\psi(x) < \frac{\operatorname{I\!E}_{\operatorname{I\!P}}[g(Z_0)]}{\inf g(z)} e^{-Rx} ,$$

$$\psi(x;\underline{y}x) < \frac{\mathbb{E}_{\mathbb{I}\!\!P}[g(Z_0)]}{\inf g(z)} \mathrm{e}^{-\underline{R}x} ,$$

$$\psi(x) - \psi(x;\overline{y}x) < \frac{\mathbb{E}_{\mathbb{I}\!\!P}[g(Z_0)]}{\inf g(z)} \mathrm{e}^{-\overline{R}x} ,$$

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where \underline{R} and \overline{R} are defined as before.

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If g(z) is bounded away from zero we obtain as before

$$\psi(x) < \frac{\operatorname{I\!E}_{\operatorname{I\!P}}[g(Z_0)]}{\inf g(z)} e^{-Rx} ,$$

$$\psi(x;\underline{y}x) < \frac{\mathbb{E}_{\mathbb{P}}[g(Z_0)]}{\inf g(z)} e^{-\underline{R}x} ,$$
$$\mathbb{E}_{\mathbb{P}}[g(Z_0)] = \overline{c}$$

$$\psi(x) - \psi(x; \bar{y}x) < \frac{\operatorname{ID}\left[g(Z_0)\right]}{\inf g(z)} e^{-R_x},$$

where <u>R</u> and \overline{R} are defined as before. If $\underline{y} < y_0 = 1/\theta'(R)$ $(\overline{y} > Y_0)$ then $\underline{R} > R$ $(\overline{R} > R)$.

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The Cramér–Lundberg Approximation

If we further suppose that $\{Z_t\}$ is Harris recurrent, then there is a constant C such that

 $\lim_{x\to\infty}\psi(x){\rm e}^{Rx}=C\;.$

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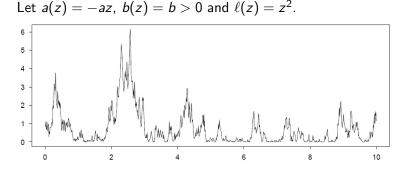
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Ornstein–Uhlenbeck Intensities



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The function g(z)

Trying $g(z) = \kappa e^{kz^2}$ we find

$$k = \frac{a - \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2b^2} ,$$

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The function g(z)

Trying $g(z) = \kappa e^{kz^2}$ we find

$$k = \frac{a - \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2b^2} ,$$

$$heta(r) = rac{a - \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2} - cr \; .$$

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The function g(z)

Trying
$$g(z) = \kappa \mathrm{e}^{k z^2}$$
 we find

$$k = \frac{a - \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2b^2} ,$$

$$heta(r) = rac{a - \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2} - cr \; .$$

Choosing κ such that $\operatorname{I\!E}[g(Z_0)] = 1$ gives

$$\kappa = \sqrt{\frac{a + \sqrt{a^2 - 2b^2(M_Y(r) - 1)}}{2a}}$$

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The Diffusion Under Q_r

For the diffusion we find

$$\begin{aligned} \tilde{\mathfrak{A}}f(z) &= \frac{-\kappa e^{kz^2}az + b^2 2kz \kappa e^{kz^2}}{\kappa e^{kz^2}} f'(z) + \frac{1}{2}b^2 f''(z) \\ &= -(a - 2kb^2)zf'(z) + \frac{1}{2}b^2 f''(z). \end{aligned}$$

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The Sparre–Andersen Risk Model

Let B(x) be the ladder height distribution. Then for $\rho = \psi(0)$

$$\psi(x) = (1-\rho) \sum_{n=1}^{\infty} \rho^n (1-B^{*n}(x)) .$$

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The Sparre–Andersen Risk Model

Let B(x) be the ladder height distribution. Then for $\rho = \psi(0)$

$$\psi(x) = (1-\rho) \sum_{n=1}^{\infty} \rho^n (1-B^{*n}(x)) .$$

If B(x) is subexponential we find

$$\psi(x) \sim \frac{\rho}{1-\rho}(1-B(x)) \; .$$

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The Sparre–Andersen Risk Model

Let B(x) be the ladder height distribution. Then for $\rho = \psi(0)$

$$\psi(x) = (1-\rho) \sum_{n=1}^{\infty} \rho^n (1-B^{*n}(x)) .$$

If B(x) is subexponential we find

$$\psi(x)\sim \frac{\rho}{1-\rho}(1-B(x)).$$

In the Sparre–Andersen Risk model $1 - B(x) \sim \mu^{-1} \int_x^\infty (1 - G(y)) \, dy$, provided $\mu^{-1} \int_0^x (1 - G(y)) \, dy$ is subexponential.

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The Björk–Grandell Model

If for large initial capital ruin occurs then the surplus will not recover until the next change of the intensity provided $\sigma_k L_k$ is not too large.

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The Björk–Grandell Model

If for large initial capital ruin occurs then the surplus will not recover until the next change of the intensity provided $\sigma_k L_k$ is not too large.

Suppose there is a $\delta > 0$ such that $\mathbb{E}[e^{\delta \sigma L}] < \infty$. Then

$$\psi(x) \sim \frac{\operatorname{I\!E}[\sigma L]}{c \operatorname{I\!E}[\sigma] - \operatorname{I\!E}[\sigma L] \mu} \int_x^\infty (1 - G(y)) \, \mathrm{d}y \; ,$$

provided $\mu^{-1} \int_0^x (1 - G(y)) \, dy$ is subexponential.

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The Markov-Modulated Model

Let H(x) be a distribution such that the integrated tail is a subexponential distribution. Suppose that $(1 - G_i(x)) \sim a_i(1 - H(x))$ and that $a = \sum_{i=1}^{\mathscr{I}} \pi_i L_i a_i > 0$. Then we find

$$\psi_i(x) \sim rac{\mathsf{a}}{\mathsf{c} - \sum_{i=1}^{\mathscr{I}} \pi_i L_i \mu_i} \int_x^\infty (1 - \mathsf{H}(y)) \, \mathrm{d} y \; .$$

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Ornstein–Uhlenbeck Intensity

For Ornstein–Uhlenbeck intensities regeneration is fast. Thus also here

$$\begin{split} \psi(x) &\sim \frac{b^2/(2a)}{c - \mu b^2/(2a)} \int_x^\infty (1 - G(y)) \, \mathrm{d}y \\ &= \frac{b^2}{2ac - \mu b^2} \int_x^\infty (1 - G(y)) \, \mathrm{d}y \; . \end{split}$$

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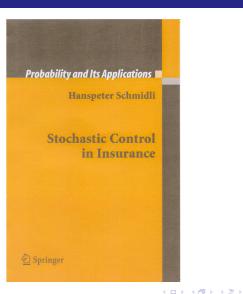
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Proportional Reinsurance

Consider the classical model and suppose the insurer can buy proportional reinsurance with retention level $\{b_t\}$. Then

$$X_t^b = x + \int_0^t c(b_s) \,\mathrm{d}s - \sum_{i=1}^{N_t} b_{T_i-} Y_i \;,$$

where c(b) is the part of the premium left for the insurer. We look for $\psi(x) = \inf_b \mathbb{P}[\inf_t X_t^b < 0]$.

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 $\psi(x)$ solves then

$$\inf_{b} c(b)\psi'(x) + \lambda \Big[\int_0^\infty \psi(x-by) \,\mathrm{d}G(y) - \psi(x)\Big] = 0 \;.$$

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The Hamilton–Jacobi–Bellman Equation

 $\psi(x)$ solves then

$$\inf_{b} c(b)\psi'(x) + \lambda \Big[\int_0^\infty \psi(x-by) \, \mathrm{d}G(y) - \psi(x) \Big] = 0 \; .$$

The optimal strategy is $b_t = b^*(X_t^b)$, where b(x) minimises the left hand side of the equation.

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Optimal Investment

Suppose the insurer can invest into a risky asset

$$Z_t = Z_0 \exp\{(m - \frac{1}{2}\sigma^2) + \sigma W_t\}$$
.

Using a strategy $\{A_t\}$ the surplus fulfils

 $\mathrm{d}X_t^{\mathcal{A}} = (c + mA_t)\,\mathrm{d}t + \sigma A_t\,\mathrm{d}W_t - \mathrm{d}S_t\;.$

We again want to minimise $\psi(x) = \inf_A \mathbb{P}[\inf_t X_t^A < 0]$.

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Then $\psi(x)$ fulfils

$$0 = \inf_{A} \frac{1}{2} \sigma^{2} A^{2} \psi''(x) + (c + mA) \psi'(x)$$
$$+ \lambda \left[\int_{0}^{\infty} \psi(x - y) \, \mathrm{d}G(y) - \psi(x) \right]$$

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Then $\psi(x)$ fulfils

$$0 = \inf_{A} \frac{1}{2} \sigma^{2} A^{2} \psi''(x) + (c + mA) \psi'(x)$$

+ $\lambda \left[\int_{0}^{\infty} \psi(x - y) \, \mathrm{d}G(y) - \psi(x) \right]$
= $c \psi'(x) - \frac{m^{2} \psi'(x)^{2}}{\sigma^{2} \psi''(x)} + \lambda \left[\int_{0}^{\infty} \psi(x - y) \, \mathrm{d}G(y) - \psi(x) \right].$

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The Hamilton–Jacobi–Bellman Equation

Then $\psi(x)$ fulfils

$$0 = \inf_{A} \frac{1}{2} \sigma^{2} A^{2} \psi''(x) + (c + mA) \psi'(x)$$

+ $\lambda \left[\int_{0}^{\infty} \psi(x - y) dG(y) - \psi(x) \right]$
= $c \psi'(x) - \frac{m^{2} \psi'(x)^{2}}{\sigma^{2} \psi''(x)} + \lambda \left[\int_{0}^{\infty} \psi(x - y) dG(y) - \psi(x) \right].$

The optimal strategy is $A_t = A^*(X_t^A) = -m\psi'(X_t^A)/(\sigma^2\psi''(X_t^A))$.

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The Hamilton–Jacobi–Bellman Equation

Then $\psi(x)$ fulfils

$$0 = \inf_{A} \frac{1}{2} \sigma^{2} A^{2} \psi''(x) + (c + mA) \psi'(x)$$

+ $\lambda \left[\int_{0}^{\infty} \psi(x - y) \, \mathrm{d}G(y) - \psi(x) \right]$
= $c \psi'(x) - \frac{m^{2} \psi'(x)^{2}}{\sigma^{2} \psi''(x)} + \lambda \left[\int_{0}^{\infty} \psi(x - y) \, \mathrm{d}G(y) - \psi(x) \right].$

The optimal strategy is $A_t = A^*(X_t^A) = -m\psi'(X_t^A)/(\sigma^2\psi''(X_t^A))$. Note that $\psi(x)$ becomes convex.

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Optimal Reinsurance: the Lundberg Bound

For a constant reinsurance strategy $b_t = b$ the Lundberg exponent R(b) is the solution to

 $\lambda(M_Y(br)-1)-c(b)r=0.$

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Lundberg Bound

For a constant reinsurance strategy $b_t = b$ the Lundberg exponent R(b) is the solution to

$$\lambda(M_Y(br)-1)-c(b)r=0$$

Let $R = \sup_{b} R(b) = R(b^{*})$.

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Lundberg Bound

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Cramér–Lundberg Approximations

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 .

Let $R = \sup_{b} R(b) = R(b^*)$. Then R is the solution to

$$\inf_b \lambda(M_Y(br)-1)-c(b)r=0.$$

We find $\psi(x) < e^{-Rx}$.

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Minimal Ruin Probabilities

Cramér-Lundberg Approximations

Optimal Reinsurance: the Martingale

Let
$$\theta(b) = \lambda(M_Y(bR) - 1) - c(b)R \ge 0$$
. Then

$$M_t = \exp\left\{-R(X_t - x) - \int_0^t \theta(b^*(X_s)) \,\mathrm{d}s\right\}$$

is a martingale.

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Change of Measure

Define the measure $Q_R[A] = \mathbb{E}[M_T; A]$. Then $\{X_t\}$ is a PDMP with

intensity: $\lambda M_Y(Rb(x))$,

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Change of Measure

Define the measure $Q_R[A] = \mathbb{E}[M_T; A]$. Then $\{X_t\}$ is a PDMP with

intensity:

claim size distribution:

$$\begin{split} &\lambda M_Y(Rb(x)),\\ &\frac{1}{M_Y(b(x)R)}\int_0^y \mathrm{e}^{Rb(x)z}\,\mathrm{d}G(z), \end{split}$$

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Change of Measure

Define the measure $Q_R[A] = \mathbb{E}[M_T; A]$. Then $\{X_t\}$ is a PDMP with

intensity:

claim size distribution:

premium rate:

$$\begin{split} &\lambda M_Y(Rb(x)),\\ &\frac{1}{M_Y(b(x)R)}\int_0^y \mathrm{e}^{Rb(x)z}\,\mathrm{d}G(z),\\ &c(b(x)). \end{split}$$

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Cramér–Lundberg Approximations

Optimal Reinsurance: the Change of Measure

Define the measure $Q_R[A] = \mathbb{E}[M_T; A]$. Then $\{X_t\}$ is a PDMP with

intensity:

claim size distribution:

premium rate:

$$\begin{split} &\lambda M_Y(Rb(x)),\\ &\frac{1}{M_Y(b(x)R)}\int_0^y \mathrm{e}^{Rb(x)z}\,\mathrm{d}G(z),\\ &c(b(x)). \end{split}$$

The drift of the process is negative.

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Minimal Ruin Probabilities

Cramér-Lundberg Approximations

Optimal Reinsurance: the Ruin Probability

The function $\psi(x)$ can be expressed as

$$\psi(x) = \operatorname{I\!E}_R \left[\exp \left\{ R X_\tau + \int_0^\tau \theta(b^*(X_s)) \, \mathrm{d}s \right\} \right] \mathrm{e}^{-Rx} \geq \operatorname{I\!E}_R [\mathrm{e}^{RX_\tau}] \mathrm{e}^{-Rx} \; .$$

As for the classical model it follows that $\psi(x) \ge \underline{C}e^{-Rx}$.

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Optimal Reinsurance: the lim sup

Let $f(x) = \psi(x)e^{Rx}$ and $g(x) = -\psi'(x)e^{-Rx} = Rf(x) - f'(x)$.

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Optimal Reinsurance: the lim sup

Let $f(x) = \psi(x)e^{Rx}$ and $g(x) = -\psi'(x)e^{-Rx} = Rf(x) - f'(x)$. Then, using the definition of R and the optimality of b(x)

$$\begin{split} \int_0^x (g(x-z)-g(x))(1-G(z/b^*)) \mathrm{e}^{Rz} \, \mathrm{d}z \\ &\geq \int_x^\infty (1-G(z/b^*)) \mathrm{e}^{Rz} \, \mathrm{d}z g(x) - \delta(0)(1-G(x/b^*)) \mathrm{e}^{Rx} \; . \end{split}$$

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Optimal Reinsurance: the lim sup

Let $f(x) = \psi(x)e^{Rx}$ and $g(x) = -\psi'(x)e^{-Rx} = Rf(x) - f'(x)$. Then, using the definition of R and the optimality of b(x)

$$\int_0^x (g(x-z) - g(x))(1 - G(z/b^*)) e^{Rz} dz$$

$$\geq \int_x^\infty (1 - G(z/b^*)) e^{Rz} dz g(x) - \delta(0)(1 - G(x/b^*)) e^{Rx}.$$

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We conclude that for x large enough, g(x) stays close to $\limsup g(x)$ on an arbitrary long interval.

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Optimal Reinsurance: the lim sup

Let $f(x) = \psi(x)e^{Rx}$ and $g(x) = -\psi'(x)e^{-Rx} = Rf(x) - f'(x)$. Then, using the definition of R and the optimality of b(x)

$$\int_0^x (g(x-z) - g(x))(1 - G(z/b^*)) e^{Rz} dz$$

$$\geq \int_x^\infty (1 - G(z/b^*)) e^{Rz} dz g(x) - \delta(0)(1 - G(x/b^*)) e^{Rx}.$$

We conclude that for x large enough, g(x) stays close to lim sup g(x) on an arbitrary long interval. Thus also for x large enough, f(x) stays close to lim sup f(x) on an arbitrary long interval.

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: the Cramér–Lundberg Approximation

Let $\xi = \limsup f(z)$. Choose $\beta > 0, \varepsilon > 0$ and x_0 , such that $f(x) > \xi - \varepsilon$ for $x \in [x_0 - \beta, x_0]$. Then for $T = \inf\{t : X_t < x_0\}$ $f(x) = \mathbb{E}_R \Big[f(X_T) \exp \Big\{ \int_0^T \theta(b^*(X_s)) \, \mathrm{d}s \Big\} \Big]$

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Cramér–Lundberg Approximations

Optimal Reinsurance: the Cramér–Lundberg Approximation

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$$f(x) = \mathbb{E}_{R}\left[f(X_{T})\exp\left\{\int_{0}^{T}\theta(b^{*}(X_{s})) ds\right\}\right]$$

$$\geq \mathbb{E}_{R}[f(X_{T})] \geq (\xi - \varepsilon)(1 - \delta) .$$

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Optimal Reinsurance: the Cramér–Lundberg Approximation

Let $\xi = \limsup f(z)$. Choose $\beta > 0, \varepsilon > 0$ and x_0 , such that $f(x) > \xi - \varepsilon$ for $x \in [x_0 - \beta, x_0]$. Then for $T = \inf\{t : X_t < x_0\}$

$$f(x) = \mathbb{E}_R \Big[f(X_T) \exp \Big\{ \int_0^T \theta(b^*(X_s)) \, \mathrm{d}s \Big\} \Big]$$

$$\geq \mathbb{E}_R [f(X_T)] \geq (\xi - \varepsilon)(1 - \delta) \; .$$

Therefore, $\lim f(x) = \liminf f(x) = \xi$.

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Reinsurance: Asymptotics of the Strategy

We have $\liminf f'(x) = \liminf (Rf(x) - g(x)) = 0$.

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Minimal Ruin Probabilities

Cramér-Lundberg Approximations

Optimal Reinsurance: Asymptotics of the Strategy

We have $\liminf f'(x) = \liminf (Rf(x) - g(x)) = 0$. From the HJB equation we find that for x large enough

 $c(b^*(x))f'(x) < -\xi\theta(b^*(x)) + \varepsilon$.

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Optimal Reinsurance: Asymptotics of the Strategy

We have $\liminf f'(x) = \liminf (Rf(x) - g(x)) = 0$. From the HJB equation we find that for x large enough

$$c(b^*(x))f'(x) < -\xi\theta(b^*(x)) + \varepsilon$$
.

Thus $\limsup f'(x) = 0$.

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Cramér–Lundberg Approximations

Optimal Reinsurance: Asymptotics of the Strategy

We have $\liminf f'(x) = \liminf (Rf(x) - g(x)) = 0$. From the HJB equation we find that for x large enough

$$c(b^*(x))f'(x) < -\xi\theta(b^*(x)) + \varepsilon$$
.

Thus $\limsup f'(x) = 0$. Therefore $\theta(b^*(x)) \to 0$. If b^* is unique, then $\lim b^*(x) = b^*$.

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The Classical Theory		
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Minimal Ruin Probabilities

Cramér-Lundberg Approximations

Optimal Investment: the Lundberg Bound

For a constant investment strategy $A_t = A$ let R(A) be the Lundberg coefficient, that is the solution to

 $\lambda(M_Y(r) - 1) - (c + mA)r + \frac{1}{2}A^2\sigma^2r^2 = 0.$

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 $\lambda(M_Y(r) - 1) - (c + mA)r + \frac{1}{2}A^2\sigma^2r^2 = 0.$

Let $R = \sup_A R(A)$. Then R solves

$$0 = \inf_{A} \lambda(M_{Y}(r) - 1) - (c + mA)r + \frac{1}{2}A^{2}\sigma^{2}r^{2}$$
$$= \lambda(M_{Y}(r) - 1) - cr - \frac{m^{2}}{2\sigma^{2}}.$$

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Optimal Investment: the Lundberg Bound

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$$= \lambda(M_{Y}(r) - 1) - cr - \frac{m^{2}}{2\sigma^{2}}.$$

Note that $R = R(A^*)$ for $A^* = m/(\sigma^2 R)$.

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Probabilities

The Classical Theory			Minimal Ruin Probabilities
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Optimal Investment: the Lundberg Bound

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= $\lambda(M_{Y}(r) - 1) - cr - \frac{m^{2}}{2\sigma^{2}}$.

Note that $R = R(A^*)$ for $A^* = m/(\sigma^2 R)$. We have $\psi(x) < e^{-Rx}$.

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Investment: the Cramér-Lundberg Approximation

Analogously to before we get

 $\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\xi$

for some $\xi \in (0,1)$,

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Minimal Ruin Probabilities

Cramér–Lundberg Approximations

Optimal Investment: the Cramér-Lundberg Approximation

Analogously to before we get

 $\lim_{x\to\infty}\psi(x)\mathrm{e}^{Rx}=\xi$

for some $\xi \in (0,1)$, and

 $\lim_{x\to\infty}A^*(x)=A^*\;.$

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Heavy Tails

- The Classical Theory
 - The Cramér–Lundberg Model
 - The Sparre–Andersen Model
- 2 Generalisations
 - The Björk–Grandell Model
 - The Markov-Modulated Model
 - Cox Models
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 - The Classical Models
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- 4 Minimal Ruin Probabilities
 - Introduction
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The	Classical Th	eory

Heavy Tails 00 0000 Minimal Ruin Probabilities

Optimal Investment: Subexponential Claims

Suppose that $G \in S^*$; i.e., that

$$\lim_{x \to \infty} \int_0^x \frac{(1 - G(z))(1 - G(x - z))}{1 - G(x)} \, \mathrm{d}z = 2\mu \; .$$

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Cox Risk Processes and Ruin

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Heavy Tails

Minimal Ruin Probabilities

Optimal Investment: Subexponential Claims

Suppose that $G \in S^*$; i.e., that

$$\lim_{x \to \infty} \int_0^x \frac{(1 - G(z))(1 - G(x - z))}{1 - G(x)} \, \mathrm{d}z = 2\mu \; .$$

Let $\kappa = 2\sigma^2 \lambda / m^2$. Assume that

$$\lim_{y\to\infty}\ell(y)=\lim_{y\to\infty}\frac{G'(y)}{1-G(y)}=0.$$

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Minimal Ruin Probabilities

Heavy Tails

Optimal Investment: the Function g(x)

Let
$$g(x) = -\psi'(x)/(1 - G(x))$$
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Minimal Ruin Probabilities

Heavy Tails

Optimal Investment: the Function g(x)

Let $g(x) = -\psi'(x)/(1 - G(x))$. Then

$$-\frac{m^2}{2\sigma^2} \frac{g(x)}{\ell(x) - \frac{g'(x)}{g(x)}} - cg(x) + \lambda\delta(0) + \lambda \int_0^x g(x-y) \frac{(1 - G(x-y))(1 - G(y))}{1 - G(x)} \, dy = 0.$$

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Minimal Ruin Probabilities

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Optimal Investment: the Function g(x)

Let
$$g(x) = -\psi'(x)/(1 - G(x))$$
. Then

$$-\frac{m^2}{2\sigma^2} \frac{g(x)}{\ell(x) - \frac{g'(x)}{g(x)}} - cg(x) + \lambda\delta(0) + \lambda \int_0^x g(x-y) \frac{(1 - G(x-y))(1 - G(y))}{1 - G(x)} \, dy = 0.$$

It follows, as expected, that $\lim_{x\to\infty} g(x) = 0$.

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Optimal Investment: the Asymptotics

From the HJB equation we find that

$$\lim_{x\to\infty}\frac{\psi'(x)^2}{\psi''(x)(1-G(x))}=\frac{2\sigma^2\lambda}{m^2}=\kappa\;.$$

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Integration shows

$$\psi(x) \sim \kappa \int_x^\infty \frac{1}{\int_0^y \frac{1}{1 - G(z)} \, \mathrm{d}z} \, \mathrm{d}y \; .$$

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From the HJB equation we find that

$$\lim_{x\to\infty}\frac{\psi'(x)^2}{\psi''(x)(1-G(x))}=\frac{2\sigma^2\lambda}{m^2}=\kappa\;.$$

Integration shows

$$\psi(x) \sim \kappa \int_x^\infty \frac{1}{\int_0^y \frac{1}{1 - G(z)} \mathrm{d}z} \mathrm{d}y \; .$$

By tail equivalence the results holds for all $G \in S^*$.

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Optimal Investment: Simpler Asymptotics

Suppose that $G \in \mathsf{MDA}(\exp\{-x^{-\alpha}\})$. Then

$$\psi(x) \sim rac{\kappa(lpha+1)}{lpha}(1-\mathcal{G}(x))$$
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Optimal Investment: Simpler Asymptotics

Suppose that $G \in MDA(exp\{-x^{-\alpha}\})$. Then

$$\psi(x) \sim rac{\kappa(lpha+1)}{lpha} (1-\mathcal{G}(x))$$
 .

Suppose that $G \in \mathsf{MDA}(\exp\{-e^{-x}\})$. Then

 $\psi(x) \sim \kappa(1 - G(x))$.

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Optimal Investment: Asymptotics of A(x)

We obtain the behaviour of A(x)

$$A(x) \sim rac{m}{\sigma^2} \int_0^x rac{1-G(x)}{1-G(z)} \,\mathrm{d}z \;.$$

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$$A(x) \sim rac{m}{\sigma^2} \int_0^x rac{1-G(x)}{1-G(z)} \,\mathrm{d}z \;.$$

If $G \in \mathsf{MDA}(\exp\{-x^{-\alpha}\})$, then

$$\lim_{x\to\infty}\frac{A(x)}{x}=\frac{m}{\sigma^2(\alpha+1)},$$

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Optimal Investment: Asymptotics of A(x)

We obtain the behaviour of A(x)

$$\mathsf{A}(x) \sim \frac{m}{\sigma^2} \int_0^x \frac{1 - \mathsf{G}(x)}{1 - \mathsf{G}(z)} \, \mathrm{d}z \; .$$

If $G \in \mathsf{MDA}(\exp\{-x^{-\alpha}\})$, then

$$\lim_{x\to\infty}\frac{A(x)}{x}=\frac{m}{\sigma^2(\alpha+1)},$$

If $G \in \mathsf{MDA}(\exp\{-\mathrm{e}^{-x}\})$, then

$$\lim_{x\to\infty}A(x)a(x)=\frac{m}{\sigma^2}\;.$$

In particular, A(x)/x tends to zero.

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Optimal Reinsurance: MDA(exp $\{-x^{-\alpha}\}$)

Similar methods and technical considerations yield

$$\lim_{x\to\infty}\frac{\psi(x)}{\int_x^\infty(1-G(z))\,\mathrm{d}z}=\inf_b\frac{\lambda b^\alpha}{(c(b)-\lambda\mu b)^+}\,.$$

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$$\lim_{x\to\infty}\frac{\psi(x)}{\int_x^\infty(1-G(z))\,\mathrm{d}z}=\inf_b\frac{\lambda b^\alpha}{(c(b)-\lambda\mu b)^+}\;.$$

Let b^* an argument where the inf is taken. If b^* is unique we get also

$$\lim_{x\to\infty}b(x)=b^*\;.$$

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Optimal Reinsurance: MDA(exp $\{-e^{-x}\}$)

Suppose that

$$\lim_{x \to \infty} \int_0^x \frac{(1 - G(z))(1 - G(x - z))}{1 - G(x)} \, \mathrm{d}z = 2\mu$$

and that the distribution tail 1 - G(x) is of rapid variation. Let $b_0 = \inf\{b : c(b) > \lambda \mu b\}$. Then for any $b > b_0$

$$\lim_{x\to\infty}\frac{\psi(x)}{\int_x^\infty(1-G(z/b))\,\mathrm{d} z}=0\;.$$

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$$\lim_{x\to\infty}\frac{\psi(x)}{\int_x^\infty(1-G(z/b))\,\mathrm{d} z}=0\;.$$

For the strategy we obtain that $\limsup_{x\to\infty} b(x) = b_0$.

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Thank you for your attention