KTH Engineering Sciences

SF3940 - Probability 7.5 hp

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Course description: Probability is the mathematical theory for studying randomness and is one of the fundamental subjects in applied mathematics.For a rigorous treatment of probability, the measure theoretic approach is a vast improvement over the arguments usually presented in undergraduate courses. This course gives an introduction to measure theoretic probability and covers topics such as the strong law of large numbers, the central limit theorem, conditional expectations, and martingales. It is expected, but not required, that students have had some exposure to measure theory prior to taking this course. Students will practice by studying applications and solving problems related to the theory.

Course literature:

1. Rick Durrett, *Probability: Theory and Examples*, 4th Edition, Cambridge Series in Statistical and Probabilistic Mathematics, 2010. ISBN 9780521765398
2. Jean Jacod and Philip Protter: *Probability Essentials*, Corrected Second Printing, Springer Verlag, 2004. ISBN 3-540-43871-8
3. Sid Resnick, *A Probability Path*, Birkhäuser Boston, 5th printing, 2005.
4. Allan Gut, *Probability: A Graduate Course*, Springer, 2005.
5. David Pollard, *A User's Guide to Measure Theoretic Probability*, Cambridge University Press, 2002.
6. Patrick Billingsley, *Probability and Measure*, 3rd Edition, Wiley.
7. Daniel Stroock, *Probability: An Analytic View*, 2nd Edition, Cambridge University Press, 2011.
8. Olav Kallenberg, *Foundations of Modern Probability*, 2nd Edition, Springer, 2002.

Format: The course will consist of bi-weekly discussion meetings (not standard lectures) where students present and discuss the material as well as some weekly exercises. The topic for each meeting is given below. Any of the books mentioned above can be used. The last two books are quite technical and perhaps not as accessible as the first six.

1. Measure and Integration (sigma-field, measure, integration)
2. Random variables, Expected Value, Independence
3. The Law of Large Numbers (Borel-Cantelli, 0-1 Laws, Applications)
4. The Central Limit Theorem (Generating Functions, Weak Convergence, Applications)
5. Random Walks (Recurrence, Transience)
6. Conditional Expectation (Radon-Nikodym Theorem, existence, properties)
7. Martingales (Martingale Convergence, Applications)

Aim: After completing the course students are expected to

- explain the foundations of probability in the language of measure theory,
- state the strong law of large numbers and give an outline of its proof
- have a basic knowledge of other 0-1 laws in probability
- have a working knowledge of weak convergence, characteristic functions, and the central limit theorem,
- give examples and applications of the strong law of large numbers and the central limit theorem,
- explain the concepts of recurrence and transience of random walks,
- explain the content of the Radon-Nikodym theorem and give an outline of its proof,
- explain the concept of conditional expectation, its properties and applications
- give an introduction to discrete time martingales and the martingale convergence theorem
- give examples and applications of martingales
- be able to solve basic problems related to the theory

Examination: The examination will be done as a combination of homework and oral exam.

Schedule

1. Fri 25 Jan 13-15, Rm 3733

   Introductory lecture. No assignments

2. Fri 15 Feb 13-15, Rm 3733

   Measure and integration.

3. Fri 22 Feb 13-15, Rm 3733

   Random variables, expectation, independence

4. Fri 15 Mar 13-15, Rm 3733

   The Strong Law of Large Numbers, 0-1 laws, Borel-Cantelli Lemma

5. We 27 Mar 15-17, Rm 3733

   Weak convergence, characteristic functions, the Central Limit Theorem

6. Fri 12 Apr 15-17, Rm 3733

   Random walks, recurrence, transience

7. Fri 3 May 10-12, Rm 3733

   Conditional expectation, the Radon-Nikodym theorem

8. Fri 17 May 10-12, Rm 3733

   Martingales, martingale convergence, applications of martingales