MODELS OF STOCHASTIC ADAPTATION

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References

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Somewhat related references

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1 Young's model

- In class (and hand-out): Elements of Markov Chain Theory

- Adaptive play with finite memory m and sample size k

  - the unperturbed process

  - the perturbed process

- The invariant distribution $\mu^{\varepsilon} \to \mu^{*}$ as $\varepsilon \to 0$

- A finite normal-form game $G = (I, S, u)$ has *property NDBR* (non-degenerate best replies) if, for every player $i \in I$ and pure strategy $h \in S_i$, the set

$$B_{ih} = \{x \in \square(S) : h \in \beta_i(x)\}$$

is either empty or has a non-empty (relative) interior. This is a generic property of finite games.

Proposition 1.1 (Theorem 7.2. in Young, 1998) *Let G be a finite game with the NDBR property. If k/m is sufficiently small, the unperturbed process converges with probability one to a minimal CURB set. In the limit as $\varepsilon \longrightarrow 0$, the perturbed process places probability one on the minimal CURB sets that have minimal stochastic potential (typically a unique such set).*

- Revisit examples:

Example 1 ("Coordination"):

	L	R
T	$2, 2$	$0, 0$
B	$0, 0$	$1, 1$

Example 2 ("Matching Pennies"):

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Example 3 (Unique and strict NE):

	L	C	R
T	$7, 0$	$2, 5$	$0, 7$
M	$5, 2$	$3, 3$	$5, 2$
B	$0, 7$	$2, 5$	$7, 0$

Example 4: (Risk dominance vs. Pareto dominance):

	A	B
A	$4, 4$	$0, 2$
B	$2, 0$	$3, 3$

1.1 CURB sets

- Basu and Weibull (1991)

- Let G be a finite NF game with mixed-strategy extension $\tilde{G}$.

- An intermediate approach between NE and rationalizability

- Strategy-profile sets that are "closed under rational behavior"

Definition 1.1 *A* **retract** *is a subset* $X \subset \square(S)$ *such that* $X = \times_{i \in N} X_i$ *for non-empty, closed and convex subsets* $X_i \subset \Delta(S_i)$

Definition 1.2 *A retract* X *is* **closed under rational behavior (CURB)** *if* $\tilde{\beta}(X) \subset X$

- $\square(S)$ is a CURB retract

- A singleton set $\{x\}$, for x strict NE, too

- Look for minimal CURB retracts!

Proposition 1.2 *If a CURB retract X is minimal, then $X = \square(T)$ for non-empty sets $T_i \subset S_i$*

- All of this can be done directly in terms of pure strategies

- For any set $X \subset \square(S)$, let $\beta(X) = \cup_{x \in X} \beta(x)$

Definition 1.3 *A set $T = \times_{i=1}^{n} T_i$ is* **closed under rational behavior (CURB)** *if $\beta[\square(T)] \subset T$*

Proposition 1.3 (Ritzberger-Weibull) *Let T be a CURB set. For each component C of $\square^{NE}$, either $C \cap \square(T) = \varnothing$ or $C \subset \square(T)$. Moreover, $\square(T)$ contains at least one essential component of $\square^{NE}$ and a proper equilibrium.*

Next lecture: Repeated games

References: Fudenberg and Maskin (1986) and Abreu, Dutta and Smith (1994)

THE END