KTH Mathematics

Cover times, sign-dependent random walks, and maxima

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Stockholm 2003

Ph. D. Thesis
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Akademisk avhandling som med tillstånd av Kungl Tekniska Högskolan framlägges till offentlig granskning för avläggande av filosofie doktorsexamen fredagen den 31 oktober 2003 kl 14.00 i Kollegiesalen, Administrationsbyggnaden, Kungl Tekniska Högskolan, Valhallavägen 79, Stockholm.

ISBN 91-7283-588-5 • TRITA-MAT-2003-MS-02 • ISSN 1401-2278 • ISRN KTH/MAT/DA 03/06-SE

Abstract

This thesis consists of four independent papers. The first three papers are closely connected while the fourth paper treats a slightly different problem.

In the first paper, results for cover times of simple random walks are generalized to hold for random walks with independent and identical exponentially distributed holding times. Transform techniques are used to derive the distributions. The transform for the density function of the cover time is appropriately scaled and a limit distribution is obtained. The distributions and the convergence to a limit distribution are graphically illustrated.

In the second paper the model is generalized to a sign-dependent random walk. A sign-dependent random walk is a simple random walk where the one-step transition probabilities can be different on the positive and negative half-line. The transform for the probability (density) function of the first passage time is derived, both for a sign-dependent random walk with constant holding times (probability generating function) and with independent and identical exponentially distributed holding times (Laplace transform). Some extensions of the first passage time are studied. The transform of the density function for the first passage time is scaled and a limit distribution is obtained. The Laplace transforms are numerically inverted and the distributions are illustrated by graphs.

The first passage time is used in the third paper to obtain transition probabilities for a sign-dependent random walk (also by transforms). After suitable scaling, weak convergence for a sign-dependent random walk to a certain diffusion is shown. In some special cases the transform for the limit distribution is possible to invert analytically. In the other cases numerical inversion is used.

The fourth paper concerns the number of maxima points in a three-dimensional cube where points are randomly placed. The main result is an exact explicit formula for the variance of the number of maxima. Previous formulas for the variance only contain the leading term in an asymptotic formula which give misleading estimates.

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