KTH Mathematics

Gibbs Measures and Phase Transitions in Potts and Beach Models

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Doctoral Thesis
Stockholm, Sweden 2004

Akademisk avhandling som med tillstånd av Kungl Tekniska högskolan framlägges till offentlig granskning för avläggande av teknologie doktorsexamen fredagen den 24 sep 2004 i Kollegiesalen, Administrationsbyggnaden, Kungl Tekniska högskolan, Valhallavägen 79, Stockholm.

TRITA-MAT-04-MS-10 • ISSN 1401-2286 • ISRN KTH/MAT/DA--04/05--SE • ISBN 91-7283-849-3

Abstract

The theory of Gibbs measures belongs to the borderland between statistical mechanics and probability theory. In this context, the physical phenomenon of phase transition corresponds to the mathematical concept of non-uniqueness for a certain type of probability measures.

The most studied model in statistical mechanics is the celebrated Ising model. The Potts model is a natural extension of the Ising model, and the beach model, which appears in a different mathematical context, is in certain respects analogous to the Ising model. The two main parts of this thesis deal with the Potts model and the beach model, respectively.

For the q-state Potts model on an infinite lattice, there are $q+1$ basic Gibbs measures: one wired-boundary measure for each state and one free-boundary measure. For infinite trees, we construct "new" invariant Gibbs measures that are not convex combinations of the basic measures above. To do this, we use an extended version of the random-cluster model together with coupling techniques. Furthermore, we investigate the root magnetization as a function of the inverse temperature. Critical exponents to this function for different parameter combinations are computed.

The beach model, which was introduced by Burton and Steif, has many features in common with the Ising model. We generalize some results for the Ising model to the beach model, such as the connection between phase transition and a certain agreement percolation event. We go on to study a q-state variant of the beach model. Using random-cluster model methods again we obtain some results on where in the parameter space this model exhibits phase transition. Finally we study the beach model on regular infinite trees as well. Critical values are estimated with iterative numerical methods. In different parameter regions we see indications of both first and second order phase transition.

Keywords and phrases: Potts model, beach model, percolation, random-cluster model, Gibbs measure, coupling, Markov chains on infinite trees, critical exponent.

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