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Topics on fractional Brownian motion and regular variation for stochastic processes

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Abstract

The first part of this thesis studies tail probabilities for elliptical distributions and probabilities of extreme events for multivariate stochastic processes. It is assumed that the tails of the probability distributions satisfy a regular variation condition. This means, roughly speaking, that there is a non-negligible probability for very large or extreme outcomes to occur. Such models are useful in applications including insurance, finance and telecommunications networks. It is shown how regular variation of the marginals, or the increments, of a stochastic process implies regular variation of functionals of the process. Moreover, the associated tail behavior in terms of a limit measure is derived.

The second part of the thesis studies problems related to parameter estimation in stochastic models with long memory. Emphasis is on the estimation of the drift parameter in some stochastic differential equations driven by the fractional Brownian motion or more generally Volterra-type processes. Observing the process continuously, the maximum likelihood estimator is derived using a Girsanov transformation. In the case of discrete observations the study is carried out for the particular case of the fractional Ornstein-Uhlenbeck process. For this model Whittle's approach is applied to derive an estimator for all unknown parameters.

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