1 Failure rate

$$\lambda(t) = \frac{f(t)}{1-F(t)}$$
$$R(t) = 1 - F(t) = e^{-\int_0^t \lambda(u)du}$$

2 TTT-transform

$$H_F^{-1}(v) = \int_0^{F^{-1}(v)} (1-F(u))du, \quad 0 \leq v \leq 1$$

The TTT-plot is given by the points $\big(i/n, T(x_{(i)})/T(x_{(n)})\big)$ where $T(x_{(i)})$ is the total time on test until time $x_{(i)}$.

3 Estimates

3.1 Estimates of constant failure rate

n parallel test sockets, r=number of failures, T total time on test.

$$\widehat{\lambda} = \frac{r}{T} \text{ (ML-estimate, unbiased for Type I-censoring with replacement)}$$
$$\widehat{\lambda} = \frac{r-1}{T} \text{ (unbiased for Typ II-censoring)}$$

Confidence interval, (exact for Typ II-censoring, approximate for Typ I-censoring)

$$\left(\frac{\chi^2_{1-\alpha/2}(2r)}{2T}, \frac{\chi^2_{\alpha/2}(2r)}{2T}\right)$$

3.2 Estimates of the survivor function

$$\widehat{R}(x) = \prod_{\nu} \frac{n-\nu}{n-\nu+1} \qquad \text{Kaplan-Meier}$$
$$\widehat{R}(x) = e^{-\widehat{Z}(x)} \quad \text{där } \widehat{Z}(x) = \sum_{\nu} \frac{1}{n-\nu+1} \qquad \text{Nelson}$$

where the product and the sum are taken over ν such that $x_{(\nu)} \leq x$ are times of failures. $\widehat{Z}(x)$ is the Nelson estimate of the cumulative failure rate.

4 Measures of component importance

$$B(i) = \frac{\eta(i)}{2^{n-1}} \qquad \text{where } \eta(i) \text{ is the number of critical paths for } i$$
$$I^B(i) = \frac{\partial h(p)}{\partial p_i}$$
$$I^{CR}(i) = \frac{I^B(i)(1-p_i)}{1-h(\underline{p})}$$
$$I^{VF}(i) = \frac{P(\cup_{j=1}^{m_i} E_i)}{1-h(\underline{p})} \qquad \text{where } E_j \text{ is the event that all components in the } j:\text{th minimal cut, in which component } i \text{ is contained, fails}$$
$$I^{IP}(i) = h(1_i, \underline{p}) - h(\underline{p})$$
$$RAW(i) = \frac{1-h(0_i,\underline{p})}{1-h(\underline{p})} \qquad \text{Risk Achievement Worth}$$
$$RRW(i) = \frac{1-h(\underline{p})}{1-h(1_i,\underline{p})} \qquad \text{Risk Reduction Worth}$$

5 Renewal theory

Let $N(t)$ be the number of renewals up to time t in a renewal process and let T be the time between two renewals. Let $m = E(T)$ and $\sigma^2 = V(T)$. For large t it holds that

$$E(N(t)) \approx \frac{t}{m}$$
$$V(N(t)) \approx \frac{\sigma^2 t}{m^3}$$

$N(t)$ is approximately normally distributed

6 Associated variables

For a system with components with associated state variables it holds

$$\prod_{j=1}^{k} P(\kappa_j(\underline{X}=1)) \leq P(\Phi(\underline{X})=1) \leq \prod_{i=1}^{s} P(\rho_i(\underline{X})=1)$$
$$\max_{1\leq j\leq s} \prod_{i\in S_j} p_i \leq P(\Phi(\underline{X})=1) \leq \min_{1\leq j\leq k} \prod_{i\in K_j} p_i$$