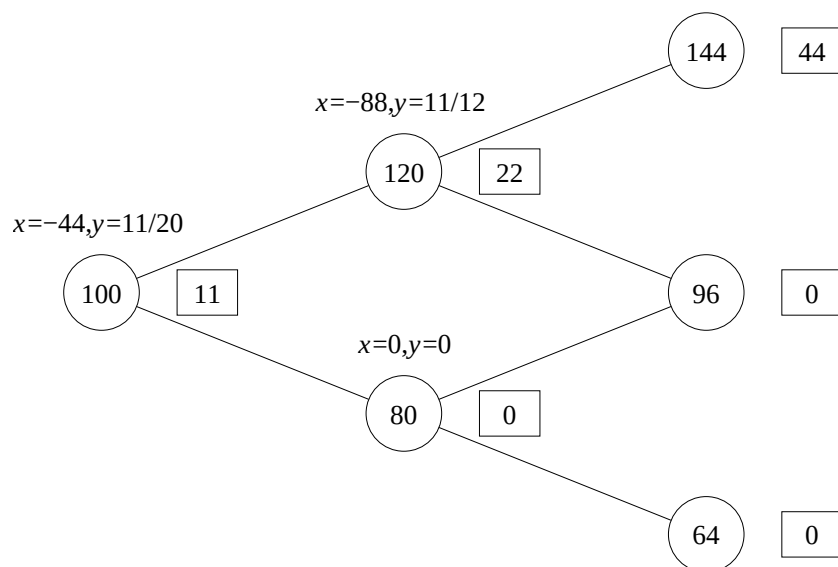


**No dividends** Call option with strike  $K = 100$ .



**General case with no dividends**

$$V_t = V_0 + \int_0^t h_u dS_u, \quad \text{if } h \text{ self-financing}$$

In the example:

$$22 = 11 + (-44) \cdot 0 + \frac{11}{20} \underbrace{(120 - 100)}_{\Delta S}$$

## General case with no dividends

$$Z_t = \frac{S_t}{B_t} \quad Q\text{-martingale}$$

In the example:

$$S_0 = E^Q \left[ \frac{S_1}{B_1} \right],$$

yields

$$100 = \frac{1}{1 + 0} [q \cdot 120 + (1 - q) \cdot 80],$$

resulting in

$$q = \frac{1}{2}.$$

## With dividends

Now let all the parameters be the same, but assume that at time  $t = 1$  the stock pays a dividend of

$$d_1 = 0.1 \cdot S_{1-}$$

Recall the jump condition

$$S_1 = S_{1-} - d_1$$

Thus if the stock price goes up we will have

$$d_1 = 0.1 \cdot 1.2 \cdot 100 = 12,$$

and

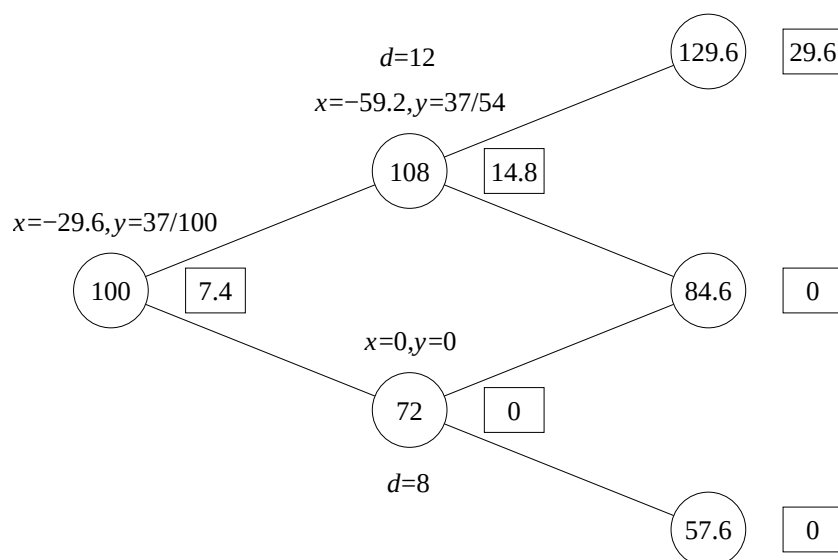
$$S_1 = 1.2 \cdot 100 - 12 = 108.$$

**Question:** What will happen to the price of the option?

**Answer:** It will become cheaper!

The reason is that the stock price at time  $t = 0$  includes the value of the dividend payment, but it is the owner of the stock who receives the dividend payment, not the owner of the option.

**With dividends** Call option with strike  $K = 100$ .



Computing the portfolio at time  $t = 0$

$$x + y(108 + 12) = -59.2 + \frac{37}{54} \cdot 108 = 14.8$$

$$x + y(72 + 8) = 0$$

**Note** that you are allowed to financing the purchase of the new portfolio using the dividend payment!

## General case with dividends

$$V_t = V_0 + \int_0^t h_u dG_u, \quad \text{if } h \text{ self-financing}$$

where  $G_t = S_t + D_t$

In the example:

$$14.8 = 7.4 + (-29.6) \cdot 0 + \frac{37}{100} (\underbrace{108 - 100}_{\Delta S} + \underbrace{12}_{\Delta D})$$

Furthermore

$$G_t^Z = \frac{S_t}{B_t} + \int_0^t \frac{1}{B(s)} dD(s) \quad Q\text{-martingale}$$

In the example:

$$S_0 = E^Q \left[ \frac{S_1}{B_1} + \frac{D_1 - D_0}{B_1} \right],$$

yields

$$100 = \frac{1}{1 + 0} [q \cdot (108 + 12) + (1 - q) \cdot (72 + 8)]$$

resulting in

$$q = \frac{1}{2}.$$