EXAMINATION IN SF2975 FINANCIAL DERIVATIVES

Date: 2014-03-19, 14:00-19:00

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Allowed technical aids: None.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Good luck!

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Problem 1

Consider a one period model with a bank account which has zero interest rate:

$$B_1 = B_0 \text{ with } B_0 = 1.$$

(a) Determine the price of the claim

$$X = \max(S_1 - 90, 0)$$

in the two state model described in Figure 1. (2 p)

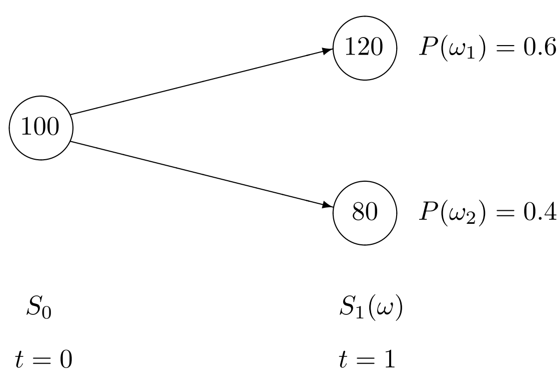

Figure 1: The two state model.

(b) Now consider the three state model in Figure 2. Show that the set of equivalent martingale measures $\mathcal{Q}$ is given by (4 p)

$$\mathcal{Q} = \{(q, 1-2q, q) \,|\, q \in (0, 1/2)\}.$$

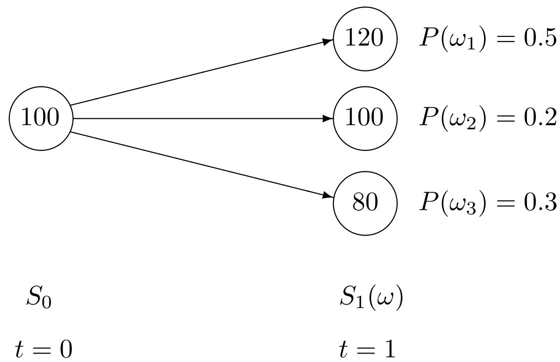

Figure 2: The three state model.

(c) Again, consider the three state model in Figure 2. Determine the set of arbitrage free prices of the claim

$$X = \max(S_1 - 90, 0).$$

(4 p)

Problem 2

Consider the standard Black-Scholes model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \text{ with } S(0) = s > 0,$$

where $r \geq 0$, $\alpha \in \mathbb{R}$ and $\sigma > 0$. Determine the arbitrage free price at time $t = 0$ of the claim

$$X = S^2(T)$$

in the following two cases:

(a) The stock pays a continuous dividend yield $\delta > 0$. (4 p)

(b) The stock pays the lump sum dividend

$$\gamma S(t-)$$

at time $t = T/2$ where $\gamma \in (0, 1)$ is a constant. (6 p)

Problem 3

Consider the following version of the Vasicek model for the short rate under an equivalent martingale measure Q with the bank account as numeraire:

$$dr(t) = -ar(t)dt + \sigma dW^Q(t).$$

Here $a \in \mathbb{R}$, $\sigma > 0$ and W^Q is a one-dimensional Wiener process under Q.

(a) Determine for every $0 \leq t \leq T < \infty$ the instantaneous forward rates $f(t, T)$. (5 p)

(b) Determine for every $t \in [0, T]$ the arbitrage free price of the T-claim

$$X = e^{r(T)}.$$

(5 p)

Problem 4

Let Q^T denote the T-forward measure for $T > 0$.

(a) Show that

$$E^{Q^T}[r(T)|\mathcal{F}_t] = f(t, T)$$

for every $t \in [0, T]$, where $r(T)$ is the short rate at time T and $f(t, T)$ is the instantaneous forward rate. (4 p)

(b) Show that the LIBOR rate $L(t; S, T)$ is a Q^T-martingale on $[0, S]$. (3 p)

(c) Let $f(t; T, X)$ denote the forward price of the T-claim X. Show that

$$f(t; T, X) = E^{Q^T}[X|\mathcal{F}_t]$$

for every $t \in [0, T]$. (3 p)

Problem 5

Consider the model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$\begin{aligned} dS_1(t) &= \alpha_1 S_1(t)dt + \sigma_1 S_1(t)dW_1(t) \text{ with } S_1(0) = s_1 > 0 \\ dS_2(t) &= \alpha_2 S_2(t)dt + \sigma_2 S_2(t)dW_2(t) \text{ with } S_2(0) = s_2 > 0, \end{aligned}$$

where $r \geq 0$, $\alpha_1, \alpha_2 \in \mathbb{R}$ and $\sigma_1, \sigma_2 > 0$ are constants, and $W = (W_1, W_2)$ is a two-dimensional Wiener process.

(a) Determine the arbitrage free price at time $t \in [0, T]$ of the T-claim

$$X = \ln\left(\frac{S_1(T)}{S_2(T)}\right).$$

(5 p)

(b) Determine the hedging portfolio of the claim in (a). (5 p)