EXAMINATION IN SF2975 FINANCIAL DERIVATIVES

Date: 2014-05-21, 14:00-19:00

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Allowed technical aids: None.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Good luck!

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Problem 1

(a) To price options on a stock, a recombining binomial tree is used. Over each time step the stock can increase in value with the factor 1.2, or decrease in value with the factor 0.8. At every time the probability of an up move is 0.55, and the moves are independent of each other. The price today ($t = 0$) of the stock is $S_0 = 100$ and there is a bank account with zero interest rate available to invest in. Calculate the price today of a European put option with maturity time $T = 4$ and strike price $K = 50$. (7 p)

(b) State and prove the put-call parity for a non-dividend paying stock. (3 p)

Problem 2

Consider the standard Black-Scholes model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \text{ with } S(0) = s > 0,$$

where $r \geq 0$, $\alpha \in \mathbb{R}$ and $\sigma > 0$. The stock pays a continuous dividend yield $\delta > 0$. Determine the arbitrage free price at time $t \in [0, T]$ of the T-claim

$$X = \mathbf{1}\left(\ln\left(S^{\beta}(T)\right) \geq K\right) = \begin{cases} 1 & \text{if} \quad \ln\left(S^{\beta}(T)\right) \geq K \\ 0 & \text{if} \quad \ln\left(S^{\beta}(T)\right) < K, \end{cases}$$

where $\beta \geq 1$ and $K > 0$ are two constants. (10 p)

Problem 3

Consider the Ho-Lee model for the short rate under an equivalent martingale measure Q with the bank account as numeraire:

$$dr(t) = \theta(t)dt + \sigma dW^Q(t).$$

Here $\sigma > 0$ is a constant, θ is a deterministic function and W^Q is a one-dimensional Wiener process under Q.

(a) Determine for every $0 \leq t \leq T < \infty$ the zero coupon bond prices $p(t,T)$ in this model. (5 p)

(b) Assume that the observed initial instantaneous forward rates are given by $f^*(0,t)$ for $t \geq 0$. Determine the function θ is order for the instantaneous forward rates at time zero to be calibrated, i.e. determine θ such that

$$f(0,t) = f^*(0,t) \text{ for every } t \geq 0.$$

(5 p)

Problem 4

A market consists of two assets: a stock denoted in the domestic currency with dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t),$$

where $\alpha \in \mathbb{R}$, $\sigma > 0$ and W is a 1-dimensional P-Wiener process, and a foreign bank account with deterministic rate $r_f > 0$. The spot exchange rate X has, under P, dynamics

$$dX(t) = \alpha_X X(t)dt + \sigma_X X(t)dW(t),$$

where $\alpha_X \in \mathbb{R}$, $\sigma_X > 0$ and W is the same Wiener process as in the dynamics of the stock price. We assume that the market is free of arbitrage.

(a) Determine the dynamics of the exchange rate X under the unique EMM Q^S where the stock is the numeraire. (5 p)

(b) Determine the implied risk-free rate in domestic currency (for example by constructing a locally risk-free portfolio of the two given assets). (5 p)

Problem 5

Consider the model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS_i(t) = \alpha_i S_i(t)dt + \sigma_i S_i(t)dW_i(t) \text{ with } S_i(0) = s_i > 0, \; i = 1, \dots, n,$$

where $r \geq 0$, $\alpha_1, \dots, \alpha_n \in \mathbb{R}$ and $\sigma_1, \dots, \sigma_n > 0$ are constants, and $W = (W_1, \dots, W_n)$ is an n-dimensional Wiener process.

(a) Determine the arbitrage free price at time $t \in [0, T]$ of the T-claim

$$X = \sum_{i=1}^{n} \sqrt{S_i(T)}.$$

(5 p)

(b) Determine the hedging portfolio of the claim in (a). (5 p)