EXAMINATION IN SF2975 FINANCIAL DERIVATIVES

Date: 2015-06-11, 14:00-19:00

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Allowed technical aids: None.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

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Problem 1

(a) Solve the PDE

$$\begin{aligned} \frac{\partial F}{\partial t} + a\frac{\partial F}{\partial x} + xF &= 0 \\ F(T,x) &= 1 \end{aligned}$$

on $[0,T]$ where $a \geq 0$ is a constant. (3 p)

(b) Determine $E^Q[r(t)]$ for $t \geq 0$ when r solves

$$dr(t) = a(r(t) - b)dt + \sigma dW^Q(t),$$

where $a, \sigma > 0$, $b \in \mathbb{R}$ and W^Q is a one-dimensional Wiener process under Q. (3 p)

(c) Let Y be the arithmetic Brownian motion

$$dY(t) = \alpha dt + \sigma dW(t) \text{ with } Y(0) = y \in \mathbb{R},$$

where $\alpha, \sigma \in \mathbb{R}$. Determine for $0 \leq T_1 \leq T_2$

$$E\left[Y(T_1) \cdot Y(T_2)\right]$$

(4 p)

Problem 2

Consider the standard Black-Scholes model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \text{ with } S(0) > 0,$$

where $r \geq 0$, $\alpha \in \mathbb{R}$ and $\sigma > 0$ and the stock pays the constant dividend yield $\delta > 0$. Determine the arbitrage free price $\Pi(t; X)$ at time $t \in [0, T]$ of the T-claim

$$X = (S(T) + 1)^3 .$$

(10 p)

Problem 3

Consider the following model for the forward rates under the objective measure P:

$$df(t, T) = \alpha(t, T)dt + \sigma(t, T)dW(t).$$

Here W is a two-dimensional Wiener process under P and the volatility function is given by

$$\sigma(t, T) = \left[\begin{array}{cc} \sigma_{11}e^{-a(T-t)} & \sigma_{21} + \sigma_{22}(T-t) \end{array} \right],$$

where $\sigma_{11}, \sigma_{21}, \sigma_{22}, a > 0$. We assume that the market model is free of arbitrage. Determine the dynamics of $f(t, T)$ for $0 \leq t \leq T < \infty$ under the martingale measure Q. (10 p)

Problem 4

Let Q^T denote the T-forward measure for $T > 0$.

(a) Show that the LIBOR rate $L(t; S, T)$ is a Q^T-martingale on $[0, S]$. (3 p)

(b) Determine $E^{Q^T}[r(T)|\mathcal{F}_t]$ when r satisfies

$$dr(t) = adt + \sigma dW^Q(t),$$

where $a, \sigma > 0$ and W^Q is a Wiener process under the martingale measure where the bank account is the numeraire. (7 p)

Problem 5

Consider the model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS_i(t) = \alpha_i S_i(t)dt + \sigma_i S_i(t)dW_i(t) \text{ with } S_i(0) = s_i > 0,\ i = 1, \ldots, n,$$

where $r \geq 0$, $\alpha_1, \ldots, \alpha_n \in \mathbb{R}$ and $\sigma_1, \ldots, \sigma_n > 0$ are constants, and $W = (W_1, \ldots, W_n)$ is an n-dimensional Wiener process. Determine the arbitrage free price at time $t \in [0, T]$ of the T-claim

$$X = \prod_{i=1}^{n} S_i(T)^{b_i},$$

where $b_i > 0$, $i = 1, \ldots, n$, are constants. (10 p)