EXAMINATION IN SF2975 FINANCIAL DERIVATIVES

*Date:* 2015-10-23, 14:00-19:00

*Lecturer:* Fredrik Armerin, tel. 070-251 75 55, email: armerin@math.kth.se

*Allowed technical aids:* None.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

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## Problem 1

(a) A stock has dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t)$$

between its dividend payments. Here $W$ is a one-dimensional Wiener process, $\alpha \in \mathbb{R}$, $\sigma > 0$ and $S(0) > 0$. There is a dividend payment of

$$\delta \cdot S(T_1-)$$

at time $T_1 > 0$, where $\delta \in (0,1)$. Calculate $E[S(T)]$ for $T > T_1$. (5 p)

(b) Let $W$ be a one-dimensional Wiener process. Determine the distribution of $X(T)|\mathcal{F}_t^W$ for $0 \leq t \leq T$ when $X$ has dynamics

$$dX(t) = (a + bt)^2 \, dW(t)$$

and where $a, b > 0$. (3 p)

(c) Show that the price of a ZCB in a model which has an ATS is convex as a function of the short rate. (2 p)

## Problem 2

Consider the standard Black-Scholes model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1,$$

and stock price dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \text{ with } S(0) > 0,$$

where $r, \alpha \in \mathbb{R}$ and $\sigma > 0$.

Determine the arbitrage free price $\Pi(t; X)$ at time $t \in [0, T_0]$ of the $T$-claim

$$X = \int_{T_0}^{T} (S(u) - S(T_0))du,$$

where $T_0 < T$. (10 p)

**Problem 3**

Consider the short rate model

$$dr(t) = adt + \sigma\sqrt{t}dW^Q(t),$$

where $a \in \mathbb{R}$, $\sigma > 0$ and $W^Q$ is a one-dimensional $Q$-Wiener process.

(a) Calculate for every $0 \leq t \leq T$ the LIBOR spot rates $L(t, T)$ in this model. (5 p)

(b) Determine the arbitrage free price of the $T$-claim

$$X = r(T)^2.$$

(5 p)

**Problem 4**

Consider the model with bank account dynamics

$$dB(t) = rB(t)dt \text{ with } B(0) = 1$$

and stock price dynamics

$$\begin{aligned} dS_1(t) &= \alpha_1 S_1(t)dt + S_1(t)\big[\sigma_{11}dW_1(t) + \sigma_{12}dW_2(t)\big] \\ dS_2(t) &= \alpha_2 S_2(t)dt + S_2(t)\big[\sigma_{21}dW_1(t) + \sigma_{22}dW_2(t)\big], \end{aligned}$$

where $S_1(0), S_2(0) > 0$, $r, \alpha_1, \alpha_2 \in \mathbb{R}$ and $\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22} > 0$. Finally $W = (W_1, W_2)$ is a two-dimensional Wiener process.

(a) Determine the arbitrage free price $\Pi(t; X)$ at time $t \in [0, T]$ of the $T$-claim

$$X = \ln S_1(T) + \ln S_2(T).$$

(5 p)

(b) Determine the hedging portfolio for the claim in (a). (5 p)

## Problem 5

Consider a model with stock price dynamics

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \text{ with } S(0) > 0,$$

where $\alpha \in \mathbb{R}$, $\sigma > 0$ and $W$ is a one-dimensional Wiener process. To model a stochastic short rate, the model

$$dr(t) = (b - ar(t))dt + c\,dW(t)$$

is used. Here $a, b, c > 0$, $W$ is the same Wiener process as in the model of $S$ and the bank account has dynamics

$$dB(t) = r(t)B(t)dt \text{ with } B(0) = 1.$$

We further assume that $\sigma > 1/a$.

(a) Determine the dynamics of $S$ and $r$ under the EMM $Q$ where the bank account is the numeraire. (3 p)

(b) Fix $T > 0$ and let $Q^T$ denote the $T$-forward measure. Determine the dynamics of $S$ and $r$ under $Q^T$. (7 p)