1 Currency swaps

Another type of swap is a *fixed-for-fixed currency swap*. In such a swap principal and payments at a fixed rate of interest in one currency is exchanged for principal and payments at a fixed rate in another currency. The principal amounts are usually exchanged at the beginning and at the end of the life of the swap.
More precisely, fix equidistant times $T_0, T_1, \ldots, T_n$, principals K_d in domestic currency and K_f in foreign currency, and rates r_d and r_f expressed as simple rates for the time interval $\Delta = T_i - T_{i-1}$. Then the payments of the swap are as follows for the domestic party

- At time T_0 K_d is paid and K_f is received.
- At time T_i $c_f = K_f r_f \Delta$ is paid and $c_d = K_d r_d \Delta$ is received.
- At time T_n $c_f + K_f$ is paid and $c_d + K_d$ is received.

Pay	K_d	c_f	c_f	c_f	$c_f + K_f$
Time	T_0	T_1	T_i	T_{n-1}	T_n
Receive	K_f	c_d	c_d	c_d	$c_d + K_d$

Usually the principals are chosen so that

$$K_d = X_{T_0} K_f$$

where X_{T_0} denotes the exchange rate at time T_0 expressed as units of domestic currency per unit of foreign currency. With this choice of principals the initial exchange of principals can be left out when valuing the swap.
Also when the swap is set up the swap rates are usually set in such a way that the value of the swap is approximately zero.
Assume that $K_d = X_{T_0} K_f$, then the value of the swap where domestic currency is received and foreign currency paid is

$$\begin{aligned} \Pi_{swap}(t) &= p^d_{fixed}(t) - X(t) p^f_{fixed}(t) \\ &= \sum_{i=1}^{n} c_d p^d(t, T_i) + K_d p^d(t, T_n) - X(t) \left\{ \sum_{i=1}^{n} c_f p^f(t, T_i) + K_f p^f(t, T_n) \right\} \end{aligned}$$

where $p^d(t, T)$ are domestic zero coupon bond prices, $p^f(t, T)$ are foreign zero coupon bond prices, and $X(t)$ is the exchange rate at time t expressed as units of domestic currency per unit of foreign currency. For a concrete example of computations, see Example 7.7 in "Fundamentals of Futures and Options Markets", and Example 7.4 in "Options, Futures, and Other Derivatives" by Hull.

Example 1 This example follows Table 7.6 in "Fundamentals of Futures and Options Markets", and Table 7.8 in "Options, Futures, and Other Derivatives" by Hull.

Suppose that the following borrowing rates are available.

	USD	AUD
General Electric	5.0%	7.6%
Qantas Airways	7.0%	8.0%

In the above table you can see that interest rates are higher in Australia than in the US and that General Electric is considered to be more creditworthy than Qantas Airways since they borrow at lower rates. Also notice that the spread (the difference between the rate offered to Qantas Airways and the rate offered to General Electric) in the US is 7.0-5.0 = 2%, whereas the spread in Australia is only 8.0-7.6 = 0.4%. The difference in spreads means that there is money to be made, ideally 2.0-0.4 = 1.6%.

Suppose that General Electric wants to borrow 20 million AUD and that Qantas Airways wants to borrow 15 million USD and that the current exchange rate is 0.75 USD/AUD. Then $K_d = 15$ million USD and $K_f = 20$ million AUD and $K_d = 15 = 0.75 \cdot 20 = K_f$, so we are in a perfect position to set up a currency swap to make use of the difference in spreads. One way to set up the hedge is the following:

This swap has the effect of transforming the USD interest rate of 5% per annum to an AUD interest rate of 6.9% per annum for General Electric. So General Electric is 0.7% per annum better than it would be if the swap had not been set up. From Qantas point of view the swap transforms the AUD interest rate of 8% per annum to a USD interest rate of 6.3% and ends up 0.7% per annum better than if the swap had not been set up. The financial institution makes a gain of 1.3% on its cash flows in USD, and a 1.1% loss on its cash flows in AUD. The net gain, ignoring exchange risk, is 0.2%. Thus the net gain to all parties is 0.7+0.7+0.2 = 1.6% as expected.

Another way to set up the swap is the following:

In this case Qantas Airways is exposed to foreign exchange risk, since it pays 1.1% in AUD and 5.2% in USD.

A third option is given by

Here General Electric bears foreign exchange risk since it receives 1.1% in USD and pays 8% in AUD. Probably the first alternative would be used in practice, since the financial institution is in the best position to hedge the foreign exchange risk. □