KTH Mathematics

Exercise in SF2701 Financial Mathematics, basic course, spring 2014.

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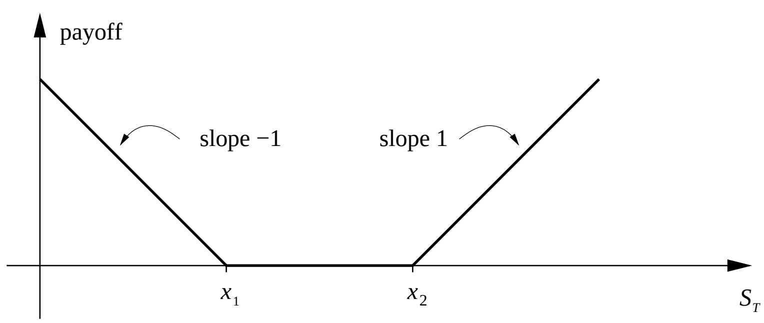
1. Suppose that you for some reason are fairly certain that there will be a large move in the stock price until time T. However you are not certain of whether the price will increase or decrease. One way to make use of your information is to buy a *strangle*, which is a T-contract with a payoff structure illustrated in the figure below.



(For the application described above today's stock price should lie between x_1 and x_2.)

Compute the price of the strangle as explicitly as possible.

Solution: Denote the payoff function by ϕ and note that

$$\phi(S_T) = \max\{x_1 - S_T, 0\} + \max\{S_T - x_2, 0\}.$$

The price of the contract is therefore

$$\begin{aligned}\Pi_t &= e^{-r(T-t)} E^Q[\max\{x_1 - S_T, 0\} + \max\{S_T - x_2, 0\} | \mathcal{F}_t] \\ &= p(t, S_t, x_1, T, r, \sigma) + c(t, S_t, x_2, T, r, \sigma).\end{aligned}$$

Here $c(t, s, K, T, r, \sigma)$ denotes the standard Black-Scholes price at time t of a European call option with exercise price K and expiry date T, when the current price of the underlying is s, the interest rate is r, and the volatility of the underlying is σ. The price of the corresponding put option is denoted by $p(t, s, K, T, r, \sigma)$.

The value of $c(t, s, K, T, r, \sigma)$ is given by the Black-Scholes formula, and using the put-call-parity

$$p(t, s, K, T, r, \sigma) = Ke^{-r(T-t)} + c(t, s, K, T, r, \sigma) - s,$$

we can also find the value of $p(t, s, K, T, r, \sigma)$ using the Black-Scholes formula. The price of the strangle is

$$\Pi_t = S_t(N[d_1(x_1)] + N[d_1(x_2)] - 1) - e^{-r(T-t)}(x_1 N[d_2(x_1)] + x_2 N[d_2(x_1)] - x_1),$$

where the expressions for d_1 and d_2 can be found in the hints at the end of the exam, except that here the value within the parenthesis refers to which strike price K should be used.