



KTH Mathematics

Homework 1 in SF2701 Financial Mathematics, basic course, spring 2017.

Due Friday April 28, 2017. Each student should hand in his or her own solutions. Please hand in a paper copy of your solutions either during class, or in the mail box at the student affairs office at Teknikringen 8 D. If you put it in the mail box, please write Camilla Landén on it. Prices should be given with two correct decimals.

1.
 - (a) Compute the price of a European call option written on a non-dividend-paying stock. The current stock price is \$100 and the volatility of the stock price is 20%. The maturity of the option is in six months and the strike price is \$100. The risk free interest rate with continuous compounding is 3% per annum. You should use a three period binomial model to price the option.
Also compute the replicating portfolio.
 - (b) The same as exercise 1a except you should price an American call option.
 - (c) Again, same as exercise 1a except this time you should price an American put option.

2.
 - (a) Same as exercise 1b except now the underlying stock will pay a dividend of 5% of the stock value in four months time. Note that you do not have to compute the replicating portfolio for this exercise.
 - (b) Again, same as exercise 1b except now the underlying stock will pay a dividend of \$5 in four months time. Note that you do not have to compute the replicating portfolio for this exercise.

Note that you are supposed to price an American **call** option which means that if early exercise is optimal, it will be so just **before** dividend payment. Make sure that you use the right stock price! (For American **put** options early exercise will always be optimal just **after** dividend payment, so that is why we have not had to think about it.)

3. A stock is expected to pay a dividend of 5% of its value in two months and in four months. The stock price is \$100, and the risk-free rate of interest is 3% per annum with continuous compounding. An investor has just taken a long position in a six-month forward contract on the stock.
- (a) What are the forward price and the initial value of the forward contract?
 - (b) Three months later the price of the stock \$95 is and the risk-free rate of interest is still 3% per annum. What are the forward price of a six-month forward contract on the stock contracted now, and the value of the long position in the forward contract contracted three months ago?
4. Compute the price of an American call option written on a futures contract. The strike price of the option and the current futures price are both \$100 and the the time to maturity of the option is six months. The risk free interest rate with continuous compounding is 3% and the the volatility of the futures price is 20%. As before, you should use a three period binomial model to price the option.
5. Quantlab exercise. Also see the following link:
<https://people.kth.se/aaurell/Teaching/SF2701-VT17/QLintro.html>
- (a) Pick three stocks from the list in the window Stock prices, click on the button Calculate. Observe the stock prices and the corresponding log-returns. Which stock has the highest volatility? Which one has the lowest?
 - (b) Calculate the standard deviation of the log-returns (see formula on p. 304 in Hull) by clicking on the Calculate button in the window Historical estimate.

Good luck!