KTH Matematik

SOLUTION TO EXAMINATION IN SF2701 FINANCIAL MATHEMATICS 2011-05-28.

Problem 1

Let $F_t = F_t^{(T)}[X/e^{R(0,T)}]$ denote the futures price at time t of $X/e^{R(0,T)}$ for delivery at T. Consider the following strategy: start with initial capital F_0 at time 0. Put F_0 in the MMA and enter into e^{r_1} long futures positions. At time 1 your balance is

$$e^{r_1}F_0 + er_1(F_1 - F_0) = e^{r_1}F_1.$$

Put this amount into MMA and increase to $e^{r_1+r_2}$ long futures positions. At time 2 your balance is

$$e^{r_1+r_2}F_1 + er_1 + r_2(F_2 - F_1) = e^{r_1+r_2}F_2.$$

Continuing in the same way until time T the final balance is

$$e^{r_1+\cdots+r_T}F_{T-1} + er_1 + \cdots + r_T(F_T - F_{T-1}) = e^{r_1+\cdots+r_T}F_T = e^{R(0,T)}X/e^{R(0,T)} = X.$$

Thus, starting from initial capital F_0 the payoff X can be generated at time T. By the law of one price the present price of X must therefore be equal to F_0.

Problem 2

One US dollar in six months can be achieved by (i) exchanging $S_0 Z_{6m}^{us}$ Indian Rupees today and put into US bank account, or (ii) entering a forward contract and buying G_0 Indian zero coupon bonds maturing in six months. The present price of the latter is $G_0 Z_{6m}^{Ind}$. Since the two prices must be equal we have

$$e^{-(r_{Ind}-r_{US})0.5} = \frac{Z_{6m}^{Ind}}{Z_{6m}^{US}} = \frac{S_0}{G_0}.$$

Then we see

$$r_{Ind} - r_{US} = 2\ln(G_0/S_0) = 6.436\%.$$

Problem 3

The tree for the bond (including the coupon at time 2) and the European call are as follows:

Table 1: Bond tree

period	1	2	3
	100.9542	105.1229	100
	102.8966	106.0789	100
		107.0446	100
			100

Table 2: European call tree

period	0	1
	21.0657	20.9542
		22.866

The price is 21.0657.

Problem 4

(a) The most plausible explanation is that there is a dividend payment between month 3 and 4.
(b) It is possible to receive one share at time 5 in two ways. The first is to enter into a forward contract with maturity 4 and hold on to the share until time 5. In this case you pay $G_0^{(4)}$ at time 4. The other is to enter into a forward contract with maturity 4 and forward rate agreement between 4 and 5 to receive $G_0^{(5)}$ at time 5. The payment of the latter at time 4 is $(Z_5/Z_4)G_0^{(5)}$. Hence,

$$G_0^{(4)} = \frac{Z_5}{Z_4} G_0^{(5)}.$$

The forward rate between 4 and 5 is then

$$f = -12 \ln(Z_5/Z_4) = -12 \ln(G_0^{(4)}/G_0^{(5)}) = 2.678\%$$

per year.
(c) Using the relation $S_0 = Z_t G_0^{(t)}$ for the Erixon share, we can compute the discount factors and the corresponding interest rates for the first three months, see table. In

months	1	2	3
forward price	92.55	92.75	92.95
discount factor	0.9978	0.9957	0.9935
interest rate (%)	2.596	2.593	2.5904

particular $Z_3 = 0.9935$.

The Noxia share seems to pay a divided between time 2 and 3. As in (b) we can use the Noxia share to compute the discount factor between 3 and 5.

$$\frac{Z_5}{Z_3} = \frac{G_0^{(3)}}{G_0^{(5)}} = 0.9952$$

The price of a zero coupon bond with maturity 5 is then

$$100 Z_5 = 100 \frac{Z_5}{Z_3} Z_3 = 98.88$$

Problem 5

A computation as on p. 71 in the Lecture Notes shows that

$$F_0 = \frac{Z(0,T)}{Z(0,t)} e^{-\frac{\sigma^2}{2}(T-t)(t-1)t}.$$